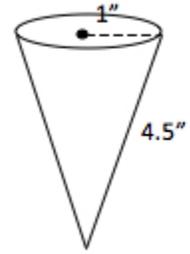


Task: Cone

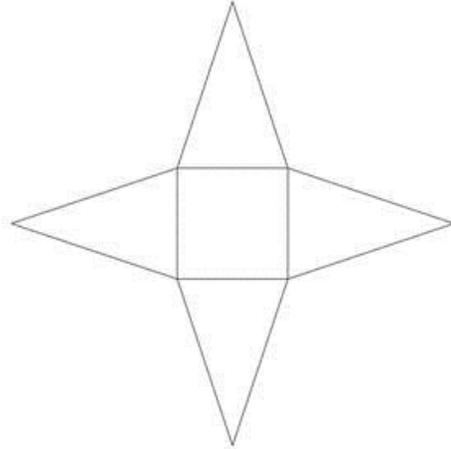
You have been hired by the owner of a local ice cream parlor to assist in his company's new venture. The company will soon sell its ice cream cones in the freezer section of local grocery stores. The manufacturing process requires that the ice cream cone be wrapped in a cone-shaped paper wrapper with a flat circular disc covering the top. The company wants to minimize the amount of paper that is wasted in the process of wrapping the cones. Use the dimensions of the ice cream cone to the right to complete the following tasks.



- Sketch a wrapper like the one described above, using the actual size of your cone. Ignore any overlap required for assembly. (Flatten the wrapper.)
- Use your sketch to help you develop an equation the owner can use to calculate the surface area of a wrapper (including the lid) for another cone given its base had a radius of length, r , and a slant height, s .
- Using measurements of the radius of the base and slant height of your cone, and your equation from the previous step, find the surface area of your cone. Show your work.
- The company has a large rectangular piece of paper that measures 100 cm by 150 cm. Estimate the maximum number of complete wrappers sized to fit your cone that could be cut from this one piece of paper. Explain your estimate.

Task: Pyramid

- a. Below is a net for a three dimensional shape:



- b. The inner quadrilateral is a square and the four triangles all have the same size and shape.
- What three dimensional shape does this net make? Explain.
 - If the side length of the square is 2 units and the height of the triangles is 3 units, what is the surface area of this shape
- c. Draw a net for a rectangular prism whose base is a one inch by one inch square and whose faces are 3 inches by 1 inch.
- Is there more than one possible net for this shape? Explain why there is only one, or provide an example of another possible net.
 - What is the surface area of the prism?

Task: Best Size Cans



The Fresha Drink Company is marketing a new soft drink.

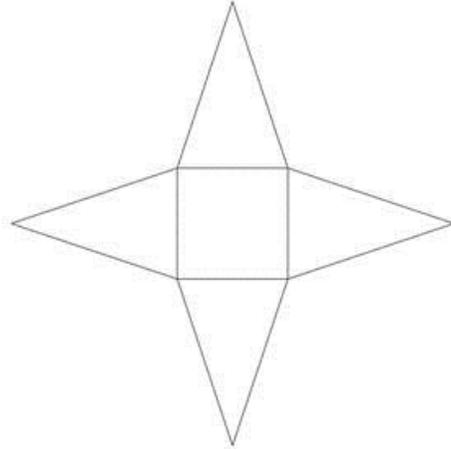
The drink will be sold in a can that holds 200 cm^3 .

In order to keep costs low, the company wants to use the smallest amount of aluminum. Find the radius and height of a cylindrical can which holds 200 cm^3 and uses the smallest amount of aluminum.

Explain your reasons and show all your calculations.

Task

a. Below is a net for a three dimensional shape:



b. The inner quadrilateral is a square and the four triangles all have the same size and shape.

- i. What three dimensional shape does this net make? Explain.
- ii. If the side length of the square is 2 units and the height of the triangles is 3 units, what is the surface area of this shape

c. Draw a net for a rectangular prism whose base is a one inch by one inch square and whose faces are 3 inches by 1 inch.

- i. Is there more than one possible net for this shape? Explain why there is only one, or provide an example of another possible net.
- ii. What is the surface area of the prism?

Task: Best Size Cans



The Fresha Drink Company is marketing a new soft drink.

The drink will be sold in a can that holds 200 cm^3 .

In order to keep costs low, the company wants to use the smallest amount of aluminum. Find the radius and height of a cylindrical can which holds 200 cm^3 and uses the smallest amount of aluminum.

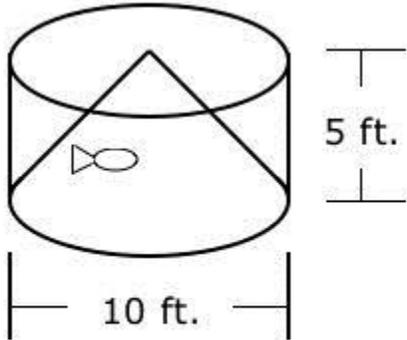
Explain your reasons and show all your calculations.

Task - Aquariums

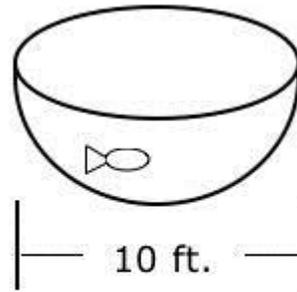
The management of an ocean life museum will choose to include either Aquarium A or Aquarium B in a new exhibit.

Aquarium A is a right cylinder with a diameter of 10 feet and a height of 5 feet. Covering the lower base of Aquarium A is an “underwater mountain” in the shape of a 5-foot-tall right cone. This aquarium would be built into a pillar in the center of the exhibit room.

Aquarium B is half of a 10-foot-diameter sphere. This aquarium would protrude from the ceiling of the exhibit room.



Aquarium A

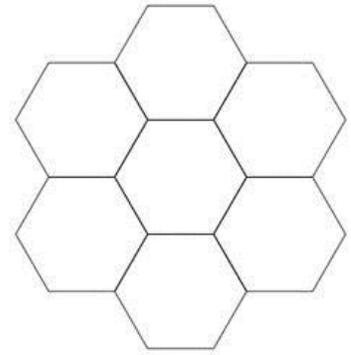


Aquarium B

- How many cubic feet of water will Aquarium A hold?
- For each aquarium, what is the area of the water's surface when filled to a height of h feet?
- Use your results from parts (a) and (b) and [Cavalieri's principle](#) to find the volume of Aquarium B.

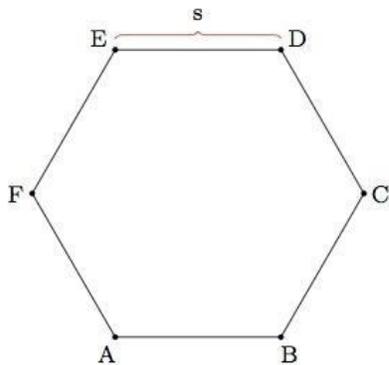
Task

Beehives are made of walls, each of the same size, enclosing small hexagonal cells where honey and pollen is stored and bees are raised. An example picture of some cells is on the right.



The only other regular polygons which can be used to tile the plane in this way are equilateral triangles and squares. This problem examines some of the mathematical advantages of the hexagonal tiling.

Suppose we let s denote the length of the walls in the hexagonal chambers. Below is an enlarged picture of a single hexagon:

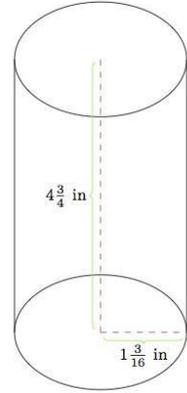


- Find the area of a regular hexagon H with side length s .
- Is the ratio of area to perimeter for a regular hexagon greater than or smaller than the corresponding ratios for an equilateral triangle and a square?
- Based on your answer to (b), why do you think it is advantageous for beehives to be built using hexagonal cells instead of triangular or square cells?

Task - Density of a Can

A cylindrical soda can is made of aluminum. It is approximately 4.75 inches high and the top and bottom have a radius of approximately $1\frac{3}{16}$ inches:

- a. Find the approximate surface area of the soda can. What assumptions do you use in your estimate?



- b. The density of aluminum is approximately 2.70 grams per cubic centimeter. If the mass of the soda can is approximately 15 grams, how many cubic centimeters of aluminum does it contain?

- c. Using the answers to (a) and (b) estimate how thick the aluminum can is.

Task: A Golden Crown?

The King asks Archimedes if his crown is made from pure gold. He knows that the crown is either pure gold or it may have some silver in it. Archimedes figures out that the volume of the crown is 125 cm^3 and that its mass is 1.8 kilograms. He also knows that 1 kilogram of gold has a volume of about 50 cm^3 and 1 kilogram of silver has a volume of about 100 cm^3 .

1. Is the crown pure gold? Explain how you know.

2. If the crown is not pure gold, then how much silver is in it? Show all your work.

Task: Hard as Nails

Tatiana is helping her father purchase supplies for a deck he is building in their backyard.

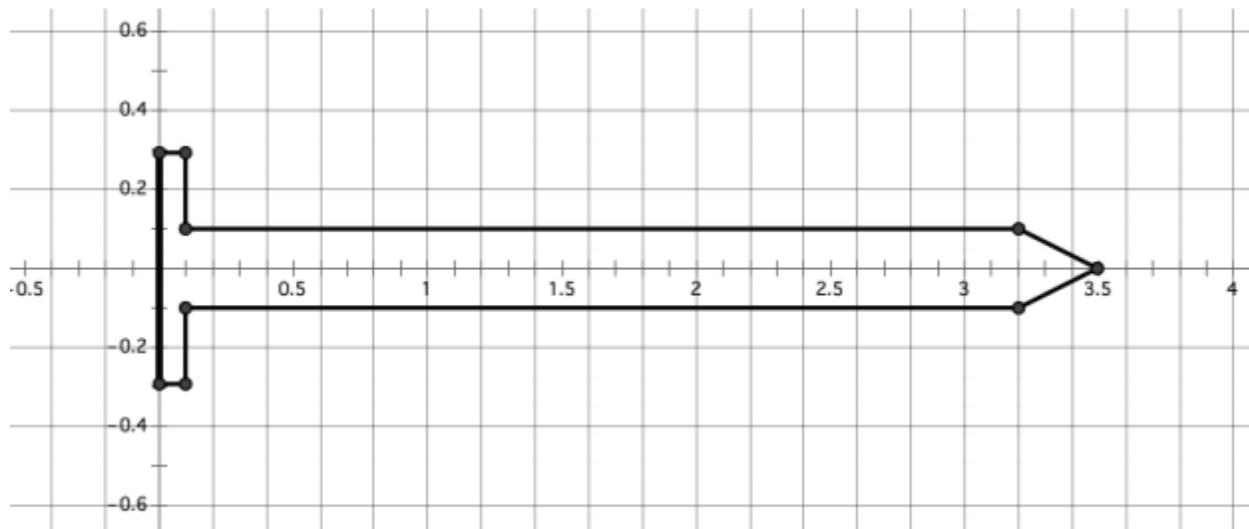
Based on her measurements for the area of the deck, she has determined that they will need to purchase 24 decking planks.

These plans will be attached to the framing joists with 16d nails. (Tatiana thinks it is strange that these nails are referred to as 16 penny nails and wonders where that way of naming nails comes from. After doing some research she has found that in the late 1700s in England the size of a nail was designated by the price of purchasing one hundred nails of that size. She doubts that her dad will be able to buy one hundred 16d nails for 16 pennies.)

Nails are sold by the pound at the local hardware store, so Tatiana needs to figure out how many pounds of 16d nails to tell her father to buy. She has gathered the following information.

- The deck requires 24 decking planks
- Each plank requires 9 nails to attach it to the framing joists
- 16d nails are made of steel that has a density of 4.65 oz/in^3
- There are 16 ounces in a pound

Tatiana has also found the following drawing of a cross section of a 16d nail. She knows she can use this drawing to help her find the volume of the nail treating it as a solid of revolution. (Note: the scale on the x and y axis is in inches)

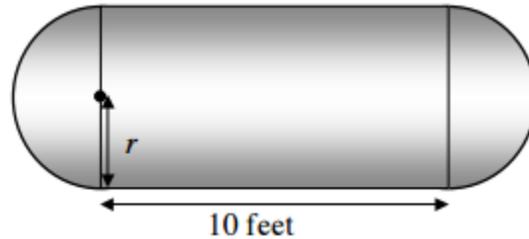


Exactly how many pounds of 16d nails does Tatiana's father need to buy? (Round your answer to the hundredth place.)

Task: Propane Tanks

People who live in isolated or rural areas have their own tanks of natural gas to run appliances like stoves, washers, and water heaters.

These tanks are made in the shape of a cylinder with hemispheres on the ends.



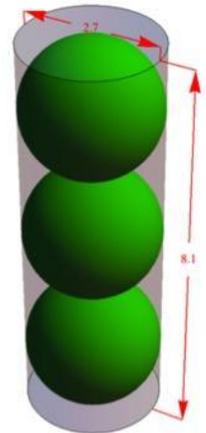
The Insane Propane Tank Company makes tanks with this shape, in different sizes. The cylinder part of every tank is exactly 10 feet long, but the radius of the hemispheres, r , will be different depending on the size of the tank.

The company want to double the capacity of their standard tank, which is 6 feet in diameter. What should the radius of the new tank be?

Explain your thinking and show your calculations.

Task - Tennis Ball

The official diameter of a tennis ball, as defined by the International Tennis Federation, is at least 2.575 inches and at most 2.700 inches. Tennis balls are sold in cylindrical containers that contain three balls each. To model the container and the balls in it, we will assume that the balls are 2.7 inches in diameter and that the container is a cylinder the interior of which measures 2.7 inches in diameter and $3 \times 2.7 = 8.1$ inches high.



- a. Lying on its side, the container passes through an X-ray scanner in an airport. If the material of the container is opaque to X-rays, what outline will appear? With what dimensions?
- b. If the material of the container is partially opaque to X-rays and the material of the balls is completely opaque to X-rays, what will the outline look like (still assuming the can is lying on its side)?
- c. The *central axis* of the container is a line that passes through the centers of the top and bottom. If one cuts the container and balls by a plane passing through the central axis, what does the intersection of the plane with the container and balls look like? (The intersection is also called a *cross section*. Imagine putting the cut surface on an ink pad and then stamping a piece of paper. The stamped image is a picture of the intersection.)
- d. If the can is cut by a plane parallel to the central axis, but at a distance of 1 inch from the axis, what will the intersection of this plane with the container and balls look like?
- e. If the can is cut by a plane parallel to one end of the can—a horizontal plane—what are the possible appearances of the intersections?
- f. A cross-section by a horizontal plane at a height of $1.35 + w$ inches from the bottom is made, with $0 < w < 1.35$ (so the bottom ball is cut). What is the area of the portion of the cross section inside the container but outside the tennis ball?
- g. Suppose the can is cut by a plane parallel to the central axis but at a distance of w inches from the axis ($0 < w < 1.35$). What fractional part of the cross section of the container is inside of a tennis ball?