Math I Unit 1: Equations, Inequalities, and Functions

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Common Core Standards

8.EE.7 Solve linear equations in one variable.
   a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where $a$ and $b$ are different numbers).
   b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

NC.M1.A-CED.1 Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.

NC.M1.A-CED.4 Solve for a quantity of interest in formulas used in science and mathematics using the same reasoning as in solving equations.

NC.M1.A-REI.1 Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning.

NC.M1.A-REI.3 Solve linear equations and inequalities in one variable.

NC.M1.F-IF.1 Build an understanding that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range by recognizing that:
   - if $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$.
   - the graph of $f$ is the graph of the equation $y = f(x)$.

NC.M1.F-IF.2 Use function notation to evaluate linear, quadratic, and exponential functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

NC.M1.F-IF.3 Recognize that recursively and explicitly defined sequences are functions whose domain is a subset of the integers, the terms of an arithmetic sequence are a subset of the range of a linear function, and the terms of a geometric sequence are a subset of the range of an exponential function.

NC.M1.F-IF.4 Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.

NC.M1.F-IF.5 Interpret a function in terms of the context by relating its domain and range to its graph and, where applicable, to the quantitative relationship it describes.

NC.M1.F-IF.6 Calculate and interpret the average rate of change over a specified interval for a function presented numerically, graphically, and/or symbolically.

MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.4 Model with mathematics.

MP.7 Look for and make use of structure.
Common Core Math 1 Unit 1  Equations, Inequalities, and Functions

Learner Objectives:
I understand that . . .

- I must use the Distributive Property and combining like terms to simplify expressions.
- Solving equations and inequalities is a process of reasoning.
- Maintaining equality is key to the process of solving equations and inequalities.
- There are strategies to clear fractions and algebraic proportions when solving multi-step equations and inequalities.
- An exact solution obtained algebraically may or may not be a practical solution in context of the problem.
- Relations and functions can be described by graphs, tables, and equations.
- There is a difference between the explicit and recursive formula.
- Increasing and decreasing refers to patterns in the y-values of a graph as read from left to right.
- Evaluating functions is a process of understanding the differences between the domain and range.
- There are multiple ways to solve equations and I must be able to justify my method using appropriate mathematical properties.
- The process of solving literal equations can be used in math and science.
- Rate of change and average rate of change can be calculated and interpreted using an equation, graph, or table.

I can...

- Use mathematical properties to justify a chosen solution method and each step in the process of solving an equation or inequality algebraically.
- Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and combining like terms.
- Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions.
- Determine how many solutions an equation has by successively transforming the equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where $a \neq b$).
- Use function notation to evaluate a given value in the domain.
- Construct models of functions using graphs, equations, and tables.
- Interpret the meaning of the independent and dependent variables in context.
- Determine if a relation is a function and justify my answer based on the definition of a function.
- Evaluate functions given inputs of their domains.
- Interpret statements that use function notation in terms of their context.
- Interpret the key features of a function, including where the function is increasing and decreasing (positive and negative) when given the function as a table, graph, and/or verbal description.
- Sketch the graph of the function showing key features given a verbal description of a relationship between two quantities.
- Determine the theoretical and practical domains of a function.
- Describe the real world meaning of the domain and range of a function.
- State the domain and range of a function from its graph.
- Calculate and interpret the average rate of change of a function over a specified interval given a table, graph, or verbal description.
- Estimate the rate of change from a graph.
- Display a graph using an appropriate scale and units.
- Generate a recursive sequence given the first term and the recursive rule.
- Determine the recursive and explicit formulas given a sequence.
- Evaluate an explicit sequence for any number of terms.

Essential Questions:

- Why is it helpful to write numbers in different formats?
- When is it appropriate to create and use an equation versus an inequality to model a given situation and/or solve a given problem?
- In what scenarios can algebraic functions be utilized to solve problems in your life?
- How can the relationship between two quantities be described or represented?
- Where in the real world can we find functions that can be modeled?
- How are the key features identified, described, and interpreted from different representations of functions
- Why are formulas important in math and science?
**Vocabulary:** Define each word and give examples and notes that will help you remember the word/phrase.

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<td>y-value</td>
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Order of Operations

Examples:

a) $3 + 6 \cdot 5 \div 3$

b) $9 - \frac{5}{8-3} + 6$

c) $150 \div (6 + 3 \cdot 8)$

Distributive Property

Examples:

a) $-7(-8 + 4r)$

b) $3(7n + 1) + 5$

c) $5 - (x + y)$
Order of Operations
Evaluate each expression.

1. 30 − 3 ÷ 3
2. (21 − 5) ÷ 8
3. 7 × 7 − (8 − 2)
4. 7 × 9 − 7 − 3 × 5

5. 4(4 ÷ 2 + 4)
6. \(\frac{1}{2}(4 + 6) - 3\)
7. (3 − 5)² + −4
8. (6 − 4) × 49 ÷ 7

9. \(\frac{45}{8(4-6)^2-7}\)
10. \(\frac{(3-1)^3}{2^2} - 17\)
11. \(\frac{43-1}{4+2} + 10\)
12. \(\frac{3}{4}(5 - 1)^2\)

Distributive Property
Simplify each expression.

13. 3(4 + 5x)
14. −2(3 + y)
15. 5(−1 − 9t)
16. 4(2x + y − 3)

17. 3³ + 4(t − 1)
18. −(−2 − n)
19. −4 + 2(5d + 6)
20. \(\frac{1}{2}(3n - 16)\)
Writing and Simplifying Algebraic Expressions

Look at each vocabulary term. In the front of the packet, write an example and any notes that will help YOU remember the definition.

- algebraic expression, coefficient, constant, equivalent expression, integers, like terms, simplify, substitution property of equality, term and variable

Writing Algebraic Expressions

Write the verbal phrase or word that mean the same thing as the algebraic symbol at the top of the column. (Note we will not use x for multiplication. Why?)

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<th>+</th>
<th>-</th>
<th>•</th>
<th>÷ or /</th>
<th>$x$ or ^</th>
<th>=</th>
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Examples: Write each phrase as an algebraic expression.

1. thirteen plus $v$ ____ 13 + $v$ ____
2. six minus $w$ ____________
3. $18$ less than the sale price ____________
4. the quotient of $n$ and $12$ ____________
5. $8$ less than the product of $25$ and a number $q$ ____________
6. $8$ times the sum of $28$ and a number $g$ ____________
7. $10$ more than a number $s$ times $5$ ____________
Combining Like Terms and Simplifying Expressions

Like terms must have the same ___________________ with the same ______________. 

<table>
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<th>Like Terms</th>
<th>Not Like Terms</th>
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* How are algebraic expressions, algebraic equations, and algebraic inequalities alike and different?

**Examples: One Variable**

Identify the coefficients and constants, then combine the like terms.

1) \(-13c + c\)  
   a. Coefficients: ___________________  
   b. Constants: ___________________  
   c. Simplify: ___________________

2) \(2x + 3x - 2 + 4x + 5\)  
   a. Coefficients: ___________________  
   b. Constants: ___________________  
   c. Simplify: ___________________

**Examples: Two or More Variables**

Identify the terms in the expression, then combine the like terms.

3) \(0.3a - b + 0.9a + 3b\)  
   a. Coefficients: ___________________  
   b. Constants: ___________________  
   c. Simplify: ___________________

4) \(8f - 2t + 3f + t\)  
   a. Coefficients: ___________________  
   b. Constants: ___________________  
   c. Simplify: ___________________

**Examples: Variables Raised to Different Exponents**

Identify the terms in the expression, then combine the like terms.

5) \(3x + 2x^2 - 2.6x + 5x^2 + 7\)  
   a. Coefficients: ___________________  
   b. Constants: ___________________  
   c. Simplify: ___________________

6) \(3a^2 - a - 7 + 5a^2 + a + 4\)  
   a. Coefficients: ___________________  
   b. Constants: ___________________  
   c. Simplify: ___________________
Examples: With Parentheses
When multiplying two factors, and at least one factor has multiple terms, use the
_______________________ property to simplify the product.

1. \(3(b+9) + 10\)
   a. Distribute: ____________________ 
   b. Simplify: ______________________

2. \(4y - 7 + 8(y+5)\)
   a. Distribute: ____________________ 
   b. Simplify: ______________________

HW: Writing Expressions: Write the following expressions in algebraic form.

3. the quotient of \(z\) and 9 ________________
4. the total of \(n\) and 40 ________________

5. the sum of 8 and \(m\) ________________
6. \(x\) divided by 5 ________________

7. the difference of \(h\) and 7 ________________
8. 23 less than \(p\) ________________

9. the product of \(g\) and 2 ________________
10. 77 plus twice \(v\) ________________

11. 9 more than \(c\) ________________
12. \(b\) minus 4 ________________

13. two times the quantity of \(r\) increased by twelve ________________

Simplifying Expressions: Identify the coefficient and constant(s) in expressions.

14. \(8x^2 + 9x - 3\)
   a) coefficient(s): ________________
   b) constant(s): ________________

15. \(17a^4 - 2a^2 + a - 1\)
   a) coefficient(s): ________________
   b) constant(s): ________________

Simplify the following expressions. Clearly show your work.

16. \(3(4x - 5)\) = __________________
17. \(-4(x - 2)\) = __________________

18. \(9 - 7(b - 10)\) = __________________
19. \(2(b - 3) - 4(2b + 2)\) = __________________

20. \(2p^4 + 3p + 12 - 18p^4 - p - 7\) = ________________
Annotating Math Word Problems - CUBES

Just like in Language Arts, we sometimes need to annotate problems to better understand them.

- **C** – Circle important numbers and variables
- **U** – Underline important words
- **B** – Box what the problem is asking you to solve
- **E** – Write an equation or expression (or inequality)
- **S** – Simplify and solve

Example:  Amanda has 2 pencils. Jacob has 3 more than 2 times the number of pencils that Amanda has. Write an expression for how many pencils Jacob has.

HW 2: Write an algebraic expression for each of the follow verbal expressions.

1. 8 minus the quotient of 15 and y
2. The absolute value of a number squared, less 8
3. The product of 8 and z plus the product of 6 and y
4. The sum of the square root of a number and 6

Write a verbal expression for each of the following algebraic expressions.

5. \( \frac{x}{4} - 17 \)
6. \( 3x + 10 \)
7. \( x^3 + 7 \)

8. Write an algebraic expression to model the following situation: The local video store charges a monthly membership fee of $5 and $2.25 per video rented.

<table>
<thead>
<tr>
<th>Videos (v)</th>
<th>Cost (c)</th>
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<tr>
<td>1</td>
<td>$7.25</td>
</tr>
<tr>
<td>2</td>
<td>$9.50</td>
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<tr>
<td>3</td>
<td>$11.75</td>
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9. Error Analysis: Describe and correct the error in the problem below:

   A student writes “the quotient of 5 and x” to describe the algebraic expression \( \frac{x}{5} \)

10. Describe a real world situation for the expression below. Be sure to identify the variable(s).

    \( b + 5 \)
Solving Equations:

When solving equations with a single variable with an exponent of 1, there are three possible solutions: a _______ solution, ______________ _______ solutions, or _____ solution.

Vocabulary Review:

Addition Property of Equality ______________________________________________________________
________________________________________________________________________________________

Subtraction Property of Equality ___________________________________________________________
__________________________________________________________________________

Multiplication Property of Equality _________________________________________________________
________________________________________________________________________________________

Division Property of Equality ______________________________________________________________
________________________________________________________________________________________

Do/Undo Charts

In the DO column, write the steps (downward) that are “done to” the x.
In the UNDO column, write the opposite operation and follow the undo column upward to undo the operations and isolate the x.

Example: \( \frac{x}{2} - 3 = -7 \)

<table>
<thead>
<tr>
<th>DO</th>
<th>UNDO</th>
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As long as equality is maintained, equations can be rewritten. To solve for a variable, it needs to be isolated and have a coefficient of 1. Typically, these steps work the best when solving for a variable.

- Simplify each side of the equation. To avoid sign errors, use the definition of subtraction to rewrite subtraction as addition.
- Use the addition property of equality and the inverse property of addition so that all of the terms with the variable you are solving for are on one side of the equation and all of the other terms are on the other side.
- If the variable’s coefficient is an integer, use the division property of equality to make the variable’s coefficient 1. If the coefficient is a fraction, use the multiplication property of equality and the inverse property of multiplication to make the variable’s coefficient 1.
- Use the substitution property of equality to check the answer.

Solve each equation for \( x \). Clearly show neat steps. Check your solution by using the substitution property of equality. If your solution is “no solution” carefully check each step of your work.

1. \( 6 - 5x = -7 \)  
2. \( 2(x - 1) - 3 = -14 - 4x \)

3. \( 8 - 2(x - 3) = 9 \)  
4. \( 5(x + 2) = 2 + 3(x + 4) - 4 \)

When solving equations, sometimes you don’t get a single value for the variable.

For what value(s) of \( x \) will this equation be true? \( 3x = 3x \)

For what value(s) of \( x \) will this equation be true? \( 2x + 1 = 2x + 1 \)

If we zero out the variable term from one side of the first equation, we get \( 0 = 0 \). When we do it with the second equation, we get \( 1 = 1 \).

The reflexive property of equality \( (a = a) \) states that these equations are true. An equation that is true for all values of the variables is called an identity. There are infinitely many solutions to an identity.

Another type of equation that does not have a single value as a solution is one that has no solution.

Consider the following equation: \( x = x + 1 \)  
For what value(s) of \( x \) would this equation be true? If we zero out the variable term from one side of the equation, we get \( 0 = 1 \). There are no solutions that will work in place of \( x \).

What do you think is the best way to check your answer when you get “no solution” or “infinitely many solutions” as your answer?
5. \( 8 - 2(3x - 7) = 5x - 11x - 3 + 5 \)

6. \( 24x - 22 = -4(1 - 6x) \)

7. \( 3(x - 5) = 3x - 18 + 3 \)

8. \( 6x + 5 - 2x = 4 + 4x + 1 \)

9. \( 8x = 3x \)

10. \( 13 - (2x + 2) = 2(x + 2) + 3x \)

11. \( 12 = -4(-6x - 3) \)

12. \( 7x - 4y + 12z + 4 = 5 - 3y + 7x - y + 12 \)

13. Create an equation that has no solution.

14. Create an equation that has infinitely many solutions.

When equations have coefficients and constants that are not integers, you can use the multiplication property of equality and the least common denominator to rewrite the equation using integers. While this is not a necessary step, it is a skill that you should master.

15. \( \frac{5}{3}x - 2x = -\frac{11}{9} \)

16. \( \frac{3}{2}x + \frac{4}{3} = -\frac{85}{9} \)

17. \( \frac{9}{5}x + \frac{3}{2}x = -\frac{33}{10} \)

18. \(-2.7 - x = 7.2\)
Declare a variable and model the situation with an equation. Solve. Check your answer with the original problem.

25. Aloysius’s cell phone plan is $29.99 per month for the first 500 minutes and $0.27 for each additional minute. His bill last month before taxes was $35.12. For how many minutes did Aloysius use his cell phone last month?

Declare a variable: Let $m =$ # of minutes used over the initial 500
Model the situation with an equation: $29.99 + 0.27m = 35.12$
Solve & check.

26. Mr. Wilson spends three-fifths of his cash at Thai Villa. On his way home, he spends another $11.17 to fill up his car with gas. When he gets home, he only has $1.83 left. How much cash did he have in his wallet when he arrived at Thai Villa?

Declare a variable: Let $c =$ the initial amount of cash in dollars
Model the situation with an equation: $c - \frac{3}{5}c - 11.17 = 1.83$
Solve & check.

Extra Practice

Solve the following equations. Some equations will have a single answer, others will have no solution, and still others will have infinite solutions.

1. $2x + 2x + 2 = 4x + 2$
2. $3(x - 1) = 2x + 9$
3. $2x + 8 = 2(x + 4)$
4. $2x - x + 7 = x + 3 + 4$
5. $-2(x + 1) = -2x + 5$
6. $4x + 2x + 2 = 3x - 7$
7. $2(x + 2) + 3x = 2(x + 1) + 1$
8. $4(x - 1) = \frac{1}{2}(x - 8)$
9. $x + 2x + 7 = 3x - 7$
Solve Word Problems

- When solving a word problem, carefully read the problem and write a variable or variable expression for each unknown. Use as few variables as possible.
- Use your variables to write an equation that models the problem.
- Use properties of equality to solve the equation.
- Check to make sure that you answered the question. Use the original problem to check your answer.

For each of the following word problems, annotate using CUBES and then
  a) declare the variable(s),  b) model the word problem with an equation, and c) solve the equation. Check your solution with the original problem.

1. After Simon donated four books to the school library, he had 28 books left. How many books did Simon have before he donated the books to the school library?

2. One day Reeva baked several dozen muffins. The next day she made 12 more muffins. If she made 20 dozen muffins in all, how many dozen did she make the first day?

3. When asked how old he was, Jerry said, “400 reduced by 4 times my age is 188.” How old is Jerry?

4. The Cooking Club made some pies to sell during lunch to raise money for a field trip. The cafeteria helped by donating three pies to the club. Each pie was cut into seven pieces and sold. There were a total of 91 pieces to sell. How many pies did the club make?

5. A health club charges a $50 initial fee plus $2 for each visit. Mary has spent a total of $144 at the health club this year. Use an equation to find how many visits she has made.

6. Find two consecutive even integers such that the sum of the larger and twice the smaller is 62.

7. Find three consecutive odd integers such that the sum of the smallest and 4 times the largest is 61.
8. The sum of two numbers is 35. Three times the larger number is equivalent to 4 times the smaller number. Find the numbers.

9. Find three consecutive integers such that the sum of twice the smallest and 3 times the largest is 126.

10. Find four consecutive odd integers who sum is 56.

11. The larger of two numbers is 1 less than 3 times the smaller. Their sum is 63. Find the numbers.

12. The sum of two numbers is 172. The first is 8 less than 5 times the second. Find the first number.

13. Find two numbers whose sum is 92, if the first is 4 more than 7 times the second.

14. The sum of three numbers is 61. The second number is 5 times the first, while the third is 2 less than the first. Find the numbers.

15. The sum of three numbers is 84. The second number is twice the first, and the third is 4 more than the second. Find the numbers.

16. An 84-meter length cable is cut so that one piece is 18 meters longer than the other. Find the length of each piece.

17. The length of a rectangle is 2 cm less than 7 times the width. The perimeter is 60 cm. Find the width and length.
18. The first side of a triangle is 7 cm shorter than twice the second side. The third side is 4 cm longer than the first side. The perimeter is 80 cm. Find the length of each side.

19. The length of a rectangle is 6 cm longer than the width. If the length is increased by 9 cm and the width by 5 cm, the perimeter will be 160 cm. Find the dimensions of the original rectangle.

20. The first side of a triangle is 8 m shorter than the second side. The third side is 4 times as long as the first side. The perimeter is 26 m. Find the length of each side.

21. A triangular sail has a perimeter of 25 m. Side a is 2 m shorter than twice side b, and side c is 3 m longer than side b. Find the length of each side.

22. The length of a rectangular field is 18 m longer than the width. The field is enclosed with fencing and divided into two parts with a fence parallel to the shorter sides. If 216 m of fencing are required, what are the dimensions of the outside rectangle?

23. Matthew is 3 times as old as Jenny. In 7 years, he will be twice as old as she will be then. How old is each now?

24. Melissa is 24 years younger than Joyce. In 2 years, Joyce will be 3 times as old as Melissa will be then. How old are they now?

25. In the Championship game, Julius scored 5 points fewer than Kareem, and Wilt scored 1 point more than twice Kareem’s points. If Wilt scored 20 points more than Julius, how many points were scored by each player?
Use CUBES to annotate each problem. Set up and solve each problem on a separate sheet of paper.

1. There are five consecutive even integers. The sum of the first and fourth integers is one-fourth of the sum of the second, third, and fifth integers. What is the product of the third and fifth integers?

2. Aloysius is 15 years older than Efrem. Two years ago, Aloysius was five years younger than three times Efrem’s age. How old will Efrem be in three years?

3. Homer has four times as many donuts as Lisa. Bart has one more donut than Lisa. Together, they have a baker’s dozen. How many donuts does Homer have to eat before he has as many donuts as Bart?

4. A rectangle’s length is one inch shorter than twice its width. Its perimeter is 19 inches. What is the rectangle’s area?

5. The supplement of an angle is 10° less than three times the angle’s complement. What is the measure of the angle?

6. An isosceles triangle’s longer side is 2 cm shorter than the sum of the other two sides. The perimeter is 18 cm long. How long is the longest side? Bonus: What is the triangle’s area?

7. Two angles of a quadrilateral are equal. The third angle is 80° less than the sum of the two equal angles. The fourth angle is 20° less than half the third angle. Find the measures of the angles in the quadrilateral.

8. The perimeter of a triangle is 93 mm. If the lengths of the sides are consecutive odds integers, how long is each side of the triangle?

9. Old MacDonald can’t remember how many sheep and chickens are on his farm. He remembers that he owns five fewer sheep than twice the number of chickens he owns. He also remembers that the sheep and chickens have a total of 100 feet. How many sheep and chickens does Old MacDonald own?

10. Aloysius first four tests scores were 93, 91, 89, and 89. His score on his fifth test lowered his test average by half of a point. What did he get on the fifth test?

The following deal with uniform motion.

11. Aloysius leaves the house and drives at an average rate of 45 miles per hour. 40 minutes later, his sister Sally leaves the house and follows the same route as her brother. She drives 55 miles per hour. How long will it take Sally to catch up to her brother?

12. Sally and Aloysius leave the library walking in opposite directions on a straight sidewalk. Sally’s average walking speed is 1.5 mph faster than her brother’s average walking speed. After 48 minutes, they are 6 miles apart. What is Sally’s average walking rate in mph?

13. The towns of Mathland and Reality are 145 miles apart. A straight road connects the two towns. Mr. Wilson leaves Reality traveling at 50 mph towards Mathland. 30 minutes later, Mr. Wilson’s evil twin brother leaves Mathland traveling at 70 mph towards Reality. How far will each of them travel before they pass each other?

14. Mr. Wilson drove to Thai Villa to pick up supper. Because of bad traffic conditions, his average speed was 30 mph. On the way home, traffic is not as bad, and he averaged 36 mph. His total driving time was 44 minutes. How long did it take Mr. Wilson to drive from his house to Thai Villa? How long did it take him to drive back home?

15. Mr. Wilson and his evil twin, Mortimer, competed in the same race. Mr. Wilson is healthy because he eats his vegetables and ran at an average rate of 15 mph. Mortimer doesn’t exercise much and eats mostly Twinkies wrapped in bacon. He “ran” at an average rate of 4 mph. Mr. Wilson’s evil twin eventually crossed the finish line, but finished 3 hours and 40 minutes after Mr. Wilson. How long did it take each of them to finish the race?
<table>
<thead>
<tr>
<th>Geometry Formulas</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Area of a Circle</strong></td>
</tr>
<tr>
<td>Example and Notes to help YOU remember:</td>
</tr>
<tr>
<td><strong>Area of a Triangle</strong></td>
</tr>
<tr>
<td>Example and Notes to help YOU remember:</td>
</tr>
<tr>
<td><strong>Circumference of a Circle</strong></td>
</tr>
<tr>
<td>Example and Notes to help YOU remember:</td>
</tr>
<tr>
<td><strong>Volume of a Cylinder or Right Prism</strong></td>
</tr>
<tr>
<td>Example and Notes to help YOU remember:</td>
</tr>
<tr>
<td><strong>Volume of a Sphere</strong></td>
</tr>
<tr>
<td>Example and Notes to help YOU remember:</td>
</tr>
<tr>
<td><strong>Volume of a Cone or Pyramid</strong></td>
</tr>
<tr>
<td>Example and Notes to help YOU remember:</td>
</tr>
</tbody>
</table>
Calculate the area of the following:

1.  
   \[ \text{Area} = \pi \times (14\text{ in})^2 \]

2.  
   \[ \text{Area} = \pi \times (80.2\text{ cm})^2 \]

Calculate the perimeter or circumference:

5.  
   \[ \text{Perimeter} = 2 \times (9\text{ mm}) + 2 \times (6\text{ mm}) + 2 \times (2\text{ mm}) \]

6.  
   \[ \text{Circumference} = \pi \times (20\text{ yd}) \]

Calculate the value of \( x \) using the given information.

7. Perimeter = 41
   \[ 3x - 7 + 2x + 5x - 2 = 41 \]

8. Perimeter = 24
   \[ x - 7 + 3x + 4x - 1 = 24 \]

9. Area = 15
   \[ 2x + 1 \times 6 = 15 \]

10. Perimeter = 18
    \[ 3x + 2 + x - 1 = 18 \]

Calculate the volumes:

11.  
    \[ \text{Volume} = \pi \times (5\text{ in}) \times (4\text{ in}) \]

12.  
    \[ \text{Volume} = \frac{4}{3} \pi \times (12\text{ mi})^3 \]

13.  
    \[ \text{Volume} = \frac{1}{3} \pi \times (11\text{ cm}) \times (11\text{ cm}) \times (11\text{ cm}) \]

14.  
    \[ \text{Volume} = \frac{1}{3} \pi \times (7\text{ mi}) \times (2\text{ mi}) \times (2\text{ mi}) \]
Sometimes you have a formula, such as something from geometry, and you need to solve for some variable other than the "standard" one. For instance, the formula for the perimeter $P$ of a square with sides of length $s$ is $P = 4s$. You might need to solve this equation for $s$, so you can substitute in a value for the perimeter and figure out the side length.

Equations with several variables are called literal equations.

* Previously, we have dealt with one-variable equations. $2 - 5(x + 1) = 12$; Solve for $x$

* What does "solve for $x$" mean?

1 – 6: Identify each formula and solve for the given variable.

1. $V = \frac{1}{3} \pi r^2 h$ Solve for $h$  
2. $V = \frac{1}{3} Bh$ Solve for $h$  

3. $V = \frac{4}{3} \pi r^3$ Solve for $r^3$.  
4. $A = \frac{1}{2} (b_1 + b_2) h$ Solve for $h$  

5. Solve #4 for $b_2$.  
6. $S = 2 \pi r^2 + 2 \pi rh$ Solve for $h$  

7. $r = \frac{a}{d^2 r}$ Solve for $a$.  

8. $x + 3y = 6$  
9. $3x - 2y = 4$  

a) Solve for $x$.  
b) Solve for $y$.  
a) Solve for $x$.  
b) Solve for $y$.  

10. Annie has a cylindrical container, but she does not know its radius or height. She does know that the radius and the height are the same and that the volume of the container is $512\pi$ cubic inches. Find the radius of Annie’s container.

11. A cone with a radius of 6 centimeters and a height of 12 centimeters is filled to capacity with liquid. Find the minimum height of a cylinder with a 4 centimeter radius that will hold the same amount of liquid.

12. The volume of a cylinder is $980\pi$ in. $^3$. The height of the cylinder is 20 in. What is the radius of the cylinder?

Bonus. $w(h - y) = w + hy$ Solve for $y$. Why? Because it’s fun!
Solve for the indicated variable:

1) \( P = IRT \) \( (T) \) 
2) \( A = 2(L + W) \) \( (W) \)

3) \( y = 5x - 6 \) \( (x) \) 
4) \( 2x - 3y = 8 \) \( (y) \)

5) \( \frac{x+y}{3} = 5 \) \( (x) \) 
6) \( y = mx + b \) \( (b) \)

7) \( ax + by = c \) \( (y) \) 
8) \( A = h(b + c) \) \( (b) \)

9) \( V = LWH \) \( (L) \) 
10) \( A = 4r^2 \) \( (r^2) \)

11) \( V = \pi r^2 h \) \( (h) \) 
12) \( 7x - y = 14 \) \( (x) \)

13) \( A = \frac{x+y}{2} \) \( (y) \) 
14) \( R = \frac{E}{I} \) \( (I) \)

15) \( x = \frac{yz}{6} \) \( (z) \) 
16) \( A = \frac{r}{2l} \) \( (L) \)

17) \( A = \frac{a+b+c}{3} \) \( (b) \) 
18) \( 12x - 4y = 20 \) \( (y) \)

19) \( x = \frac{2y-z}{4} \) \( (z) \) 
20) \( P = \frac{R-C}{N} \) \( (R) \)
Inequalities
The steps and properties for solving inequalities are similar to those involving equations.

*Review: Why does the inequality symbol get reversed when both sides of the inequality are multiplied or divided by a negative quantity?

Review: Solve, graph, and do a full check before you check your work and answers with the answer key.

1. 5 – x < 3
2. \( \frac{2}{3}x + 7 \leq -9 
3. 9 > -4x - 7
4. 5x - 3 \geq 7
5. -2x + 3 > 7
6. 0.1x + 6 \leq 2

7. Sally earns $58 per week plus a 4% commission on the seashells she sells. What must her total sales be for the week if she wants to earn at least $102?
   a) Declare a variable. Let x = the value (in $) of the seashells Sally sells by the seashore in a week
   b) Write an inequality to model the situation. 0.04x + 58 \geq 102
   c) Solve & check.

Multi-step inequalities. Solve for x.
8. 3x - 2(6x - 4) > 4 - (x + 6)
9. 6x - 4 \leq 2x + 12
10. 2x + 17 < -2(8 - x)
11. 2(x + 2) > -4 + 2x
Practice the steps for solving word problems. Use CUBES to annotate.

12. Carlos goes to the fair where it costs $5 to get in and $0.80 per ride. He has $28. How many times can he go on rides at the fair?

a) Declare a variable.  

b) Write an inequality.

c) Solve & check.

13. Wednesday morning, Kristina bought suckers for her children. She gave Linus half of the ones she bought because he had cleaned his room and made his bed in the morning. She gave Nicholas one third of the ones she bought because he made his bed, but he had not yet cleaned his room. Later in the day, she bought 3 extra suckers and gave them to Nicholas due to good behavior. However, Linus made fun of Nicholas, so she took 5 suckers away from Linus. By the end of the day, Linus had at least as many suckers as Nicholas. What is the minimum number of suckers purchased by Kristina?

a) Declare a variable.  

b) Write an inequality.

c) Solve & check.

Bonus: Because this is an inequality, we don’t know for sure how many suckers were purchased. List 3 possible amounts of suckers that Kristina could have purchased on Wednesday.
Recursive Patterns: NOW-NEXT

______________________ are lists of numbers where each term is based on the previous term or a combination of previous terms using a set pattern or rule.

Writing NOW-NEXT Rules for Sequences

Given the sequence of numbers: 3, 6, 9, 12, 15...

Step 1. State the start number. The start is the number we begin with. In this case, the start number is _____.

Step 2. Write the rule in NOW-NEXT form. In the pattern above, each number increases by _____.

This means that the next number will be NOW + ______

Therefore, the rule should be written NEXT = NOW + ______. Start = ______

Try the following examples:

Determine the rule for the sequence and write it as a NOW-NEXT equation.

Example 1. 5, 10, 15, 20...

Example 2 2, 4, 8, 16, 32....

Example 3

<table>
<thead>
<tr>
<th>Term</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>-12</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
</tr>
<tr>
<td>5</td>
<td>-48</td>
</tr>
</tbody>
</table>

Example 4

<table>
<thead>
<tr>
<th>Term</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>52</td>
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<tr>
<td>2</td>
<td>46</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>34</td>
</tr>
<tr>
<td>5</td>
<td>28</td>
</tr>
</tbody>
</table>

Example 5. Write a recursive pattern for the perimeter of the shape made up by the triangles.

<table>
<thead>
<tr>
<th>n</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>
NOW-NEXT Practice: Tables, Equations, and Graphs

1) Consider the sequence of figures below made from triangles.

![Figure 1](triangle.png) ![Figure 2](triangle_2.png) ![Figure 3](triangle_3.png) ![Figure 4](triangle_4.png)

a) Complete the table below for the first five figures.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

b) Write a NOW-NEXT equation to find the perimeter of each figure.

c) Find the perimeter of the 10th figure.

d) Which number figure has a perimeter of 51?

2) List the first six values generated by the recursive routine below. Write the routine as a NOW-NEXT equation.

\[-27.4 \text{ ENTER} \]

\[\text{Ans} + 9.2 \text{ ENTER, ENTER, \ldots} \]

3) Write a NOW-NEXT equation for each sequence. Use each equation to find the 5th term of each sequence.

a) 7.8, 3.6, -0.6, -4.8, \ldots

b) -9.2, -6.5, -3.8, -1.1, \ldots

c) 1, 3, 9, 27, \ldots

d) 36, 12, 4, \frac{4}{3}, \ldots
4) For each of the graphs below, fill in the table of values and write the NOW-NEXT equation for each relationship.

```
<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

```
<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
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<tbody>
<tr>
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```
<table>
<thead>
<tr>
<th>x</th>
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<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

5) Match the routine in the first column with the equation in the second column.

a. 2
   Ans - 0.75

b. 0.75
   Ans + 2

c. -0.75
   Ans - 2

d. -2
   Ans + 0.75

1) START = 0.75
   NEXT = NOW + 2

2) START = -0.75
   NEXT = NOW + 2

3) START = 0.75
   NEXT = NOW - 2

4) START = -0.75
   NEXT = NOW - 2

5) START = 2
   NEXT = NOW -.75

6) START = -2
   NEXT = NOW -.75

7) START = 2
   NEXT = NOW + .75

8) START = -2
   NEXT = NOW + .75
Comparing Rules: Recursive Form & Explicit Form

For each sequence below write the NOW-NEXT and INPUT-OUTPUT equation.

<table>
<thead>
<tr>
<th>Sequence</th>
<th>Term</th>
<th>Value</th>
<th>NOW-NEXT Rule</th>
<th>INPUT-OUTPUT Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example 1</td>
<td>1</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>11</td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>5</td>
<td>17</td>
<td></td>
<td></td>
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<tr>
<td>Example 2</td>
<td>1</td>
<td>52</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>46</td>
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<td></td>
<td>3</td>
<td>40</td>
<td></td>
<td></td>
</tr>
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<td>5</td>
<td>28</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Define:

Recursive Form ____________________________________________
__________________________________________________________

Explicit Form ____________________________________________
__________________________________________________________

Compare and Contrast Recursive (NOW-NEXT) and Explicit (INPUT-OUTPUT) rules.

1) What information can you get from each form?

2) When is it more advantageous to use one form vs. the other?
Our input-output machine takes a number \( x \), operates on the number, and changes it to a new number \( y \). Below are some input-output machines, the input \( x \), and the output \( y \). Can you figure out the rule for what the machine did to the input?

\[ y = x + 2 \]

1. \[
\begin{array}{c|c}
X & Y \\
7 & 2 \\
-1 & -6 \\
11 & 6 \\
\end{array}
\]

2. \[
\begin{array}{c|c}
X & Y \\
9 & 3 \\
0 & 0 \\
-6 & -2 \\
\end{array}
\]

3. \[
\begin{array}{c|c}
X & Y \\
1 & -4 \\
-2 & 8 \\
4 & -16 \\
\end{array}
\]

4. \[
\begin{array}{c|c}
X & Y \\
4 & 9 \\
6 & 13 \\
1 & 3 \\
\end{array}
\]

5. \[
\begin{array}{c|c}
X & Y \\
2 & 2 \\
5 & 11 \\
0 & -4 \\
\end{array}
\]

6. \[
\begin{array}{c|c}
X & Y \\
3 & 11 \\
-1 & -1 \\
4 & 4 \\
\end{array}
\]

7. \[
\begin{array}{c|c}
X & Y \\
2 & 3 \\
0 & -5 \\
5 & 15 \\
\end{array}
\]

8. \[
\begin{array}{c|c}
X & Y \\
4 & 6 \\
12 & 14 \\
8 & 10 \\
\end{array}
\]
Testing for Functions

In this lesson you will

● represent relationships with tables, graphs, and equations
● use the vertical line test to determine whether a relationship is a function

You have written and used many rules that transform one number into another. For example, one simple rule is “Multiply each number by 2.” You can represent this rule with a table, an equation, a graph, or a diagram.

<table>
<thead>
<tr>
<th>Table</th>
<th>Equation $y = 2x$</th>
<th>Graph</th>
<th>Mapping Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input $x$</td>
<td>Output $y$</td>
<td></td>
<td>Domain</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td></td>
<td>-14</td>
</tr>
<tr>
<td>-14</td>
<td>-28</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>9.3</td>
<td>18.6</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>$x$</td>
<td>$2x$</td>
<td></td>
<td>9.3</td>
</tr>
</tbody>
</table>

In this lesson you will learn a method for determining whether a rule is a function either by applying the definition of function to graphs, tables, and mapping diagrams.

**Vocabulary:**

Relation:

Domain:

Range:

Function:
Investigation: Testing for Functions
In this investigation we will look at different representations of relationships - tables, algebraic statements (equations or inequalities), and graphs. In each case, we will decide whether the relationship represented is a function or not.

Part I: Tables

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Table 2</th>
<th>Table 3</th>
<th>Table 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>y</td>
<td>x</td>
<td>y</td>
</tr>
<tr>
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<td>-2</td>
<td>1</td>
<td>-1</td>
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<td>1</td>
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</tr>
<tr>
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<td>7</td>
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<td>2</td>
</tr>
<tr>
<td>7</td>
<td>16</td>
<td>4</td>
<td>-2</td>
</tr>
<tr>
<td>10</td>
<td>25</td>
<td>9</td>
<td>3</td>
</tr>
</tbody>
</table>

Look at Table 1. Each input has only one output, so the relationship is a function.

In Table 2, the input values 1 and 4 each have two different possible outputs: the x-value 1 has corresponding y-values of -1 and 1, and the x-value 4 has corresponding y-values of 2 and -2. So Table 2 does not represent a function.

1. Table 3 represents a function and Table 4 does not. Explain why for each table.

Part II: Equations

<table>
<thead>
<tr>
<th>Statement 1</th>
<th>Statement 2</th>
<th>Statement 3</th>
<th>Statement 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 2x + 1 )</td>
<td>( y^2 = x )</td>
<td>( y = x^2 )</td>
<td>( y &lt; -2x + 4 )</td>
</tr>
</tbody>
</table>

Consider Statement 1, \( y = 2x + 1 \). For any x-value that you input, you multiply by 2 and then add 1. There is only one possible output value that can result for any given input value. So Statement 1 represents a function.

For Statement 2, can you think of two different y-values that correspond to a single x-value? If \( x = 4 \), y can be 2 or -2, so Statement 2 does not represent a function.

2. Statement 3 represents a function, and Statement 4 does not. Explain why for each statement.
Part III: Graphs

You can move a vertical line, such as the edge of a ruler, from left to right on a graph to determine whether the graph represents a function. If the vertical line ever intersects the graph in more than one point, you know that there is an x-value that has more than one corresponding y-value, so the graph is not a function.

Graph 1 represents a function because no vertical line will intersect the graph more than once.

For Graph 2, however, 3 of the 4 vertical lines pictured intersect the graph twice. For each of these three x-values, there are two corresponding y-values, so the graph is not a function.

3. What about Graphs 3 and 4? For each graph, determine whether it is a function or not and explain why.
The **vertical line test** helps you determine whether a relationship is a function by looking at its graph. If all possible vertical lines cross the graph only once or not at all, then the graph is a function. If even one vertical line crosses the graph more than once, the graph is not a function. That domain value will have more than one range value.

4. Use the vertical line test to determine which relationships are functions.

5. Determine whether each table of x- and y-values represents a function. Explain your reasoning.

Adapted from *Discovering Algebra: An Investigative Approach* by Murdock, Kamischke, and Kamischke
Relation & Function Practice

State the domain and range of each relation.

1. \{(5,3), (–2, 6), (6, –2), (7,3)\}  
2. \{(3,7), (–2, 8), (–5, –1)\}  
3. \{(5,7), (7,3), (–1,8), (7, –2)\}

Use the vertical-line test to determine whether or not each relation is a function.

4.  
5.  
6.  

Create a mapping diagram for 1 – 3. Is each relation a function? Explain.

7.  
8.  
9.  

10. Define the following terms: relation, domain, range, and function

Write an explicit input-output rule for each table.

11. | Input | Output |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>–3</td>
<td>1</td>
</tr>
<tr>
<td>–2</td>
<td>3</td>
</tr>
<tr>
<td>–1</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>7</td>
</tr>
</tbody>
</table>

12. | Input | Output |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>–4</td>
<td>12</td>
</tr>
<tr>
<td>–2</td>
<td>6</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>–6</td>
</tr>
</tbody>
</table>

13. | Input | Output |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>–1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>–1</td>
</tr>
<tr>
<td>1</td>
<td>–2</td>
</tr>
<tr>
<td>2</td>
<td>–3</td>
</tr>
</tbody>
</table>
Function Notation

In this lesson you will
- learn to use function notation
- use a graph to evaluate a function for various input values
- use an equation to evaluate a function for various input values

The equation \( y = 1 - 2x \) represents a function. You can use the letter \( f \) to name this function and then use function notation to express it as \( f(x) = 1 - 2x \). You read \( f(x) \) as “\( f \) of \( x \),” which means “the output value of the function \( f \) for the input value \( x \).” So, for example, \( f(2) \) is the value of \( 1 - 2x \) when \( x \) is 2, so \( f(2) = -3 \). (Note: In function notation, the parentheses do not mean multiplication.)

Not all functions are expressed as equations. Here is a graph of function \( g \). The function rule is not given, but you can still use function notation to express the outputs for various inputs. For example, \( g(0) = 3 \), \( g(4) = 6 \), and \( g(6) = 1 \). Can you find 3 integer \( x \)-values for which \( g(x) = 3 \)? Can you find one non-integer value of \( x \) for which \( g(x) = 3 \)?

Investigation: A Graphic Message

In this investigation, you will apply function notation to learn the identity of the mathematician who introduced functions. Look at the graph below.

1. What is the domain of \( y = f(x) \)?

2. What is the range of \( y = f(x) \)?

3. Use the graph to find each function value in the table. Then do the indicated operations. Show all work to the right.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(3) )</td>
<td></td>
</tr>
<tr>
<td>( f(18) + f(3))</td>
<td></td>
</tr>
<tr>
<td>( f(5) \cdot f(4) )</td>
<td></td>
</tr>
<tr>
<td>( f(15) \div f(6) )</td>
<td></td>
</tr>
<tr>
<td>( f(20) ) ( - ) ( f(0) ) ( + ) ( 6 )</td>
<td></td>
</tr>
</tbody>
</table>
Think of the numbers 1 through 26 as the letters A through Z. Find the letters that match your answers in the table to learn the mathematician’s last name.

4. Use the rules for order of operations to evaluate the expressions that involve function values. Write your answers in the table. Show all work to the right of the table.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(0) + f(3) - 1 )</td>
<td></td>
</tr>
<tr>
<td>( 5 \cdot f(9) )</td>
<td></td>
</tr>
<tr>
<td>( x ) when ( f(x) = 10 )</td>
<td></td>
</tr>
<tr>
<td>( f(9 + 8) )</td>
<td></td>
</tr>
<tr>
<td>( \frac{f(17) + f(10)}{2} )</td>
<td></td>
</tr>
<tr>
<td>( f(8 \cdot 3) - 5 \cdot f(11) )</td>
<td></td>
</tr>
<tr>
<td>( f(4 \cdot 5 - 1) )</td>
<td></td>
</tr>
<tr>
<td>( f(12) )</td>
<td></td>
</tr>
</tbody>
</table>

Find the letters that match your answers in the table to learn the mathematician’s first name.

This mathematician, who lived from 1707 to 1783, was a pioneering Swiss mathematician and physicist. He made important discoveries in calculus and graph theory. He also introduced much of the modern mathematical terminology and notation, including function notation. He is also renowned for his work in mechanics, fluid dynamics, optics, and astronomy. He spent most of his adult life in St. Petersburg, Russia and in Berlin (then a part of Prussia). He is considered to be the preeminent mathematician of the 18th Century, and one of the greatest of all time.*

* adapted from Wikipedia

5. You can use the function \( f(x) = 19.4 + 1.28x \) to approximate the wind chill temperature \( f(x) \) for a given actual temperature, \( x \), when the wind speed is 15 miles per hour. Both \( x \) and \( f(x) \) are in degrees Fahrenheit. Find \( f(x) \) for each given value of \( x \).

\[ a. \ f(-10) \quad b. \ f(0) \quad c. \ x \) when \( f(x) = 19 \quad d. \ x \) when \( f(x) = -13 \]

When you write an equation for a function, you can use any letters you want to represent the variables and the function. For example, you might use \( W(t) = 19.4 + 1.28t \) for the wind chill function discussed above.
6. Use the graph of \( y = f(x) \) at the right to answer each question. Assume each mark on the grid is one unit.

   a) What is the value of \( f(4) \)?
   b) What is the value of \( f(6) \)?
   c) For what value(s) does \( f(x) = 2 \)?
   d) For what value(s) does \( f(x) = 1 \)?
   e) How many \( x \)-values make the statement \( f(x) = 0.5 \) true?
   f) For what \( x \)-values is \( f(x) \) greater than 2?
   g) What are the domain and range shown on the graph?

7. The graph of \( y = g(x) \) below shows the temperature \( y \) outside at different times \( x \) over a 24-hour period. Use the graph to answer each question.

   a) What are the dependent and independent variables?

   b) What are the domain and range shown on the graph?

   c) Use function notation to represent the temperature at 10 hours.

   d) Use function notation to represent the time at which the temperature is 10°F.
Constant and Average Rate of Change

In this lesson you will

- estimate the rate of change from a graph.
- calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval.

Every day we deal with quantities expressed as ratios: miles per gallon of gas, cost per kilowatt of power, miles per hour that a car is travelling. When working with functions that relate two quantities such as miles and gallons or cost and kilowatts or miles and hours, we refer to these ratios as rate of change. Rate of change tells us how much one quantity is changing with respect to another quantity. For example, a speed of 60 mph tells us that a vehicle travels 60 miles for each hour it is driven.

Some rates of change are constant, and others are not. For example, if a car travels from one city to another, it does not normally travel at a constant rate. The car will speed up or slow down depending on traffic. When the rate is not constant, we often look at the average rate of change. The average rate of change tells us how much one quantity changes with respect to another quantity over a specified interval. So if the car travels 150 miles in 3 hours (rate of change), we can say that the average rate of change (or speed) for those 3 hours was 50 miles per hour. Units that have a denominator of 1 (per 1 of a unit) are called unit rates.

1 – 4: Find the rate of change. Explain what the rate of change represents.

1. Cost of Hiring Movers

<table>
<thead>
<tr>
<th>Number of Hours</th>
<th>Charge in $</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>395</td>
</tr>
<tr>
<td>4</td>
<td>490</td>
</tr>
<tr>
<td>5</td>
<td>585</td>
</tr>
</tbody>
</table>

2. Water Remaining in a Barrel

3. At a Snail’s Pace

4. The cost of 5 pounds of apples is $8.75. The cost of 8 pounds of apples is $14.

5. What are some real-world examples of constant rates of change?

6. When functions involving constant rates are graphed, what characteristics do they share?
Interpreting Graphs of Functions

In this lesson you will
- describe graphs using the words increasing, decreasing, discrete, continuous, linear, and nonlinear
- match graphs with descriptions of real-world situations
- use intervals of the domain to help you describe a function’s behavior
- write a description of a real-world relationship displayed in a graph
- draw a graph to match a description of a real-world situation

Every day we are bombarded with information, often in graph form. To analyze a graph, you have to understand how the quantities in the graph relate to each other, how they make the graph go up or down or level off. The range values in a graph can change at a constant rate or at a varying rate as the x-values of a function increase steadily.

In this lesson you’ll look at graphs that show how two real-world quantities are related.

Investigation 1: Matching Up

A function is linear if, as x changes at a constant rate, the y-values change at a constant rate. The graphs of linear functions appear as straight lines. A function is nonlinear if, as x changes at a constant rate, the function values change at a varying rate. The graphs of non-linear functions are curved. If the output of a function increases as the input values increase, the function is described as increasing. If the output of a function decreases as the input values increase, the function is described as decreasing. Some functions are both increasing and decreasing functions because they are increasing functions for some domain values and decreasing functions for other domain values. Some functions are neither increasing nor decreasing.

1. Which graphs below are linear? ______________
2. Which graphs are nonlinear? ______________
3. Which graphs are increasing? ______________
4. Which graphs are decreasing? ______________
5. List any graphs that are neither increasing nor decreasing. ____________
6. List any graphs that are both increasing and decreasing. ____________
In order to describe the relationship pictured on a graph, it often helps to break the domain into intervals where the graph is increasing, decreasing, or constant.

7. Use the intervals marked on the x-axis in the graph to the left to help you state the interval of the domain where the function is increasing or decreasing and where it is linear or nonlinear. The first interval is done for you.

**Interval 1:** In the interval $0 \leq x \leq 3$, the function is nonlinear and decreasing.

**Interval 2:**

**Interval 3:**

**Interval 4:**

---

**Investigation 2: Describing Graphs**

Consider the following scenario:

A turtle crawls steadily from its pond across the lawn. Then a small dog picks up the turtle and runs with across the lawn. The dog slows down and finally drops the turtle. The turtle rests for a few minutes after this excitement. Then a young girl comes along, picks up the turtle, and slowly carries it back to the pond.

8. Which of the graphs depicts the turtle’s distance from the pond over time? Explain. ____________________________________________

---

9. Select one of the other three graphs. Write a story that would be depicted by the graph. (You can use the turtle or another situation.)

__________________________________________________________

__________________________________________________________

__________________________________________________________
Investigation 3: Discrete vs. Continuous

Functions that have smooth graphs, with no breaks in the domain or range, are called continuous functions. Functions that are not continuous often involve integers—such as the number of trees in a yard or stories of a building. Such functions are called discrete functions. Below are some examples of discrete functions.

10. Match each description with its most likely graph. Then label the axes with the appropriate quantities.
   a) the estimated amount of radiation over time after a nuclear bomb test
   b) the number of points you earn in a contest as a function of the number of questions you answer correctly
   c) the height of a blade of grass over time
   d) the number of students who help decorate for the homecoming dance vs. the time it takes to decorate

Circle the words which could be used to describe each graph.

11. Graph 1: increasing    decreasing    linear    non-linear    continuous    discrete
12. Graph 2: increasing    decreasing    linear    non-linear    continuous    discrete
13. Graph 3: increasing    decreasing    linear    non-linear    continuous    discrete
14. Graph 4: increasing    decreasing    linear    non-linear    continuous    discrete

Adapted from Discovering Algebra: An Investigative Approach by Murdock, Kamischke, and Kamischke

Interpreting Graphs of Functions: Summary

Graphs of functions can be described as increasing (the range values increase as the domain values increase), decreasing (the range values decrease as the domain values increase), both increase and decreasing (examples: parabolas and absolute values), or neither increasing nor decreasing (a horizontal line).

Graphs can be described as linear (a straight line) or non-linear (not a straight line). The graph of a function with an average rate of change or a constant rate of change will look like a line when it is graphed. Linear functions will be covered in more detail in unit 4. If the rate of change is constantly changing, the graph will be a curve when graphed. Functions involving amounts increasing and decreasing by percentages will be covered in unit 5 (exponential functions). Functions involving falling objects affected by gravity will be covered in unit 6 (quadratic functions).

Graphs can be continuous (there are an infinite number of domain values for the function between any two domain values) or discrete (one point does not connect to the next point). Discrete functions often involve concepts with which only whole numbers or integers can be used.

Graphs of functions can be broken up into intervals based on domain values. Within domain values, the graph may have different characteristics (increasing/decreasing; linear/nonlinear; discrete/continuous). Each point on the graph is a piece of data and it tells part of the story that the graph represents. Before analyzing the data of a graph, the title and the labels of the axes should be read carefully.
Unit 1 Review #1

Solve for x.

1. \(3(4x - 9) = 6(2x - 5) + 2\)  
2. \(5 - 2(3x - 4) = 7 - 5x\)  
3. \(2x - 7 = -19\)  
4. \(3(x + 8) = 4(x + 6)\)  
5. \(-5x > 20\)  
6. \(2x - 7 > -11\)  
7. \(x - 8 \leq 12\)  
8. \(5 + 2(4 - x) + x < -7\)

Simplify.

9. \(3 - 5n + 2n - 7 - 6\)  
10. \(5 - 7(x - 3) - 2x + 4x - 3 + 4\)

How many terms are there? Identify any coefficients. Identify any constants.

11. \(5x - y - 3z - 8\)

Calculate the volume of each figure. Figure 12 is a cylinder with a smaller cylinder removed.

12. [Diagram of a cylinder with a smaller cylinder removed]  
13. [Diagram of a cylinder]  
14. [Diagram of a pyramid]

15. The volume of a cylinder is \(980\pi\) cubic inches. The height of the cylinder is 20 inches. What is the radius of the cylinder?
### Unit 1 Review #2

Solve each equation below.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \frac{2}{3} + \frac{3x}{4} = \frac{7}{12} )</td>
<td>2. ( 18x - 5 = 3(6x - 2) )</td>
</tr>
<tr>
<td>3. ( 9 + 5x = 2x + 9 )</td>
<td>4. ( -8x + 14 = -2(4x - 7) )</td>
</tr>
<tr>
<td>5. ( 2(6 + 4x) &gt; 12 - 8x )</td>
<td>6. ( 2 = -\frac{3x - (-4)}{8} )</td>
</tr>
<tr>
<td>7. ( x - \frac{1}{3} = \frac{4}{5} )</td>
<td>8. ( \frac{x - 2}{x + 5} = \frac{3}{8} )</td>
</tr>
</tbody>
</table>

Solve each equation for \( x \).

<table>
<thead>
<tr>
<th>Equation</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>9. ( 3y + 2 = 9x - 4 )</td>
<td>10. ( \frac{x + y}{z} = a )</td>
</tr>
<tr>
<td>11. ( \frac{a}{x} = \frac{b}{c} )</td>
<td>12. ( 2xy + z = 5x )</td>
</tr>
</tbody>
</table>
Write an expression.

13. 4 more than twice a number. Let $x =$ the number

14. 12 times the quantity of $x$ minus 8

15. The quotient of the sum of $x$ and 3 and 5.

Declare a variable, write an equation or inequality and calculate the solution.

16. There are 4 more boys than girls in Spanish class. The class has 38 members. How many boys and girls are there separately?

17. Find three consecutive odd integers with a sum of 45.

18. Ten less than two times a number is equivalent to the number increased by 6.

19. The low temperatures for the previous two days were 62° and 58°. What would the temperature need to be for the third day such that the average daily temperature is at least 64°?

20. Manuel is taking a job translating English instruction manuals to Spanish. He will receive $15 per page plus $100 per month. He’d like to work for 3 months during the summer and make at least $1,500. Write an inequality and find the minimum number of pages Manuel must translate in order to reach his goal.

Unit 1 Review #3
Write a NOW–NEXT statement for each sequence. Calculate the 6th term in the sequence.

1. {10, 7, 4, …} 

2. Write the first 5 numbers of the NOW-NEXT sequence. START = 2 ; NEXT = 2(NOW) + 1

3. State the relation’s domain and range. Draw a mapping diagram. {(-2, 3), (6, 0), (3, 0)}

4. Is the relation in #3 a function? If so, write down a point that could be added to the relation so that it would not be a function. If it is not, write down a point that could be removed from the relation so that it would be a function.

5. $f(x) = x^2 - 5$; Evaluate the function rule for the domain {-3, 2, a}.
Use figure 1 to answer the following questions.
6. Which method should be used to determine whether or not the relation is a function?

7. What is the domain of the function?

8. What is the range of the function?

9. Is the function increasing, decreasing, neither, or both?

10. For what domain values is the function increasing? If it is not, write “none”.

11. For what domain values is the function decreasing? If it is not, write “none”.

12. Is the function discrete or continuous? Explain.

13. Is the function linear or nonlinear? Explain.

14. Evaluate the following: \( f(0) = \ldots \); \( f(1) = \ldots \); \( f(2) = \ldots \); \( f(3) = \ldots \); \( f(4) = \ldots \); \( f(5) = \ldots \)

Bonus: \( f(0.3) = \ldots \); \( f(4.25) = \ldots \) Show and/or explain your work.

15. Evaluate the functions rules for the following domain: \{-1, 0, 1\}. Clearly show your work.
\( f(x) = 2x - 1 \)
\( g(x) = x^2 \)

16. \( f(-1) = \ldots \); \( f(0) = \ldots \); \( f(1) = \ldots \)

17. \( g(-1) = \ldots \); \( g(0) = \ldots \); \( g(1) = \ldots \) Bonus: \( g(f(0)) = \ldots \)

18. Each of three function rules was evaluated for 4 domain values. Write an explicit INPUT-OUTPUT function rule that could be used for each of the following.

\( a(3) = 2 \)
\( a(4) = 3 \)
\( a(5) = 4 \)
\( a(6) = 5 \)

OUTPUT = ______________

\( b(2) = 4 \)
\( b(3) = 8 \)
\( b(4) = 16 \)
\( b(5) = 32 \)

OUTPUT = ______________

\( c(4) = 3 \)
\( c(6) = 4 \)
\( c(8) = 5 \)
\( c(10) = 6 \)

OUTPUT = ______________