Please Tell Me in Dollars and Cents Learning Task

1. Aisha made a chart of the experimental data for her science project and showed it to her science teacher. The teacher was complimentary of Aisha's work but suggested that, for a science project, it would be better to list the temperature data in degrees Celsius rather than degrees Fahrenheit.

   a. Aisha found the formula for converting from degrees Fahrenheit to degrees Celsius: $C = \frac{5}{9}(F - 32)$.
   Use this formula to convert freezing (32°F) and boiling (212°F) to degrees Celsius.

   Comment(s):
   Students are asked to do these calculations as introductory exploration of this function. Many students likely know the corresponding Celsius temperatures; this part asks them to use the formula to find the values.

   Solution(s):
   Freezing: $C = \frac{5}{9}(32 - 32) = \frac{5}{9} \cdot 0 = 0$, ***or freezing is*** **0°C.**
   Boiling: $C = \frac{5}{9}(212 - 32) = \frac{5}{9} \cdot 180 = 5 \cdot 20 = 100$, ***or boiling is*** **100°C.**

   b. Later Aisha found a scientific journal article related to her project and planned to use information from the article on her poster for the school science fair. The article included temperature data in degrees Kelvin. Aisha talked to her science teacher again, and they concluded that she should convert her temperature data again – this time to degrees Kelvin. The formula for converting degrees Celsius to degrees Kelvin is
   $$K = C + 273.$$
   Use this formula and the results of *part a* to express freezing and boiling in degrees Kelvin.

   Comment(s):
   This part is about giving some sense of Kelvin temperatures and comprehending the meaning of the conversion formula.

   Solution(s):
   Freezing: $K = 0 + 273 = 273$, ***or freezing is*** **273°K.**
   Boiling: $K = 100 + 273 = 373$, ***or boiling is*** **373°K.**

   c. Use the formulas from *part a* and *part b* to convert the following to °K: – 238°F, 5000°F .

   Comment(s):

This part is designed to prepare students for idea of composing the two functions by having them do the two step process.

Solution(s):

Converting $-238°F$ *to* $°K$: $C = \frac{5}{9}(-238-32) = \frac{5}{9}\cdot(-270) = -150$;

$K = -150 + 273 = 123$ *Thus,* $-238°F$ *is* $123°K$.

Converting $5000°F$ *to* $°K$: $C = \frac{5}{9}(5000-32) = \frac{5}{9}\cdot 4968 = 2760$

$K = 2760 + 273 = 3033$ *Thus,* $5000°F$ *is* $3303°K$.

In converting from degrees Fahrenheit to degrees Kelvin, you used two functions, the function for converting from degrees Fahrenheit to degrees Celsius and the function for converting from degrees Celsius to degrees Kelvin, and a procedure that is the key idea in an operation on functions called ***composition of functions.***

Composition of functions is defined as follows: If f and g are functions, the ***composite function*** $f \circ g$ (read this notation as "f composed with g) is the function with the formula

$$(f \circ g)(x) = f(g(x)),$$

where x is in the domain of g and $g(x)$ is in the domain of f.

2. We now explore how the temperature conversions from *Item 1*, *part c*, provide an example of a composite function.
   a. The definition of composition of functions indicates that we start with a value, x, and first use this value as input to the function g. In our temperature conversion, we started with a temperature in degrees Fahrenheit and used the formula to convert to degrees Celsius, so the function g should convert from Fahrenheit to Celsius: $g(x) = \frac{5}{9}(x-32)$. What is the meaning of x and what is the meaning of $g(x)$ when we use this notation?

      Comment(s):
      This part and the next involve the key conceptual step of moving to function notation where all of the inputs are expressed with the variable x. This is the notation they will use when they work with the inverse of a function.

      Solution(s):
      Here "x" is a temperature in degrees Fahrenheit and "g(x)" is the corresponding temperature in degrees Celsius.

   b. In converting temperature from degrees Fahrenheit to degrees Kelvin, the second step is converting a Celsius temperature to a Kelvin temperature. The function f should give us this conversion; thus, $f(x) = x + 273$. What is the meaning of x and what is the meaning of $f(x)$ when we use this notation?

Here "x" is a temperature in degrees Celsius and "f(x)" is the corresponding temperature in degrees Kelvin.

c. Calculate $(f \circ g)(45) = f(g(45))$. What is the meaning of this number?

<u>Comment(s):</u>
This part is designed to help students build familiarity with the notation. Students will organize their work in a variety of ways; the solution below provides one example.

<u>Solution(s):</u>

$$g(45) = \frac{5}{9}(45 - 32) = \frac{5}{9} \cdot 13 = \frac{65}{9} \approx 7.2;$$

$$f(g(45)) = f\left(\frac{65}{9}\right) = \frac{65}{9} + 273 = \frac{2522}{9} \approx 280.2$$

The value of $f(g(45)) \approx 280.2$ is the temperature in °K that corresponds to 45°F.

d. Calculate $(f \circ g)(x) = f(g(x))$, and simplify the result. What is the meaning of x and what is the meaning of $(f \circ g)(x)$?

<u>Comment(s):</u>
This part is designed to introduce students to the types of calculations that they will need to do in verifying inverse functions using composition. Extensive practice in simplifying formulas created using composition is reserved for GPS Pre-Calculus, standard MM4A4, part c.

<u>Solution(s):</u>

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{5}{9}(x - 32)\right) = \frac{5}{9}(x - 32) + 273 = \frac{5}{9}x + \frac{2297}{9}$$

Here "x" is a temperature in degrees Fahrenheit and "$(f \circ g)(x)$)" is the corresponding temperature in degrees Kelvin.

e. Calculate $(f \circ g)(45) = f(g(45))$ using the formula from *part d*. Does your answer agree with your calculation from *part c*?

<u>Comment(s):</u>
This part insures that students realize that the simplified version of the composition formula eliminates the need for a two-step process and helps verify their calculations from part d.

<u>Solution(s):</u>

$$(f \circ g)(45) = \frac{5}{9}(45) + \frac{2297}{9} = \frac{225}{9} + \frac{2297}{9} = \frac{2522}{9} \approx 280.2$$

Yes, this computation gives the same answer.

f. Calculate $(g \circ f)(x) = g(f(x))$, and simplify the result. What is the meaning of *x*? What meaning, if any, relative to temperature conversion can be associated with the value of $(g \circ f)(x)$?

Comment(s):
This part is designed to make students notice the domain requirements for function composition. Here the issue is not whether the value can be calculated but instead is the issue of meaning. It brings home the point that, when we are working with functions that have real-world contexts, the contexts must "match-up" in order to form meaningful compositions.

Solution(s):

$$(g \circ f)(x) = g(f(x)) = g(x+273) = \frac{5}{9}((x+273)-32) = \frac{5}{9}(x+241)$$

Here "x" is a temperature in degrees Celsius. We cannot associate a meaning to " $(g \circ f)(x)$)" relative to temperature conversion since "f(x)" is a temperature in degrees Kelvin, but an input to the function g should be a temperature in degrees Fahrenheit.

We now explore function composition further using the context of converting from one type of currency to another.

3. On the afternoon of May 3, 2009, each Japanese yen (JPY) was worth 0.138616 Mexican pesos (MXN), each Mexican peso was worth 0.0547265 Euro (EUR), and each Euro was worth 1.32615 US dollars (USD).[1]

   a. Using the rates above, write a function *P* such that *P*(*x*) is the number of Mexican pesos equivalent to *x* Japanese yen.

   Comment(s):
   Students may need to experiment with some specific values, such as starting with 1000 yen, in order to decide how to write the formula. Alternately, writing a proportion may help:

$$\frac{P(x)\ \text{Mexican pesos}}{x\ \text{Japanese yen}} = \frac{0.138616\ \text{Mexican pesos}}{1\ \text{Japanese yen}}.$$

[1] Students may find it more interesting to look up current exchange values to use for this item and Item 9, which depends on it. There are many websites that provide rates of exchange for currency. Note that these rates change many times throughout the day, so it is impossible to do calculations with truly "current" exchange values. The values in Item 3 were found using http://www.xe.com/ucc/ .

Writing conversion functions sometimes seems backwards to students. For example, they know that 1 foot = 12 inches, but, to write a function that converts from feet to inches, the formula would be I = 12F, where I is the number of inches and F is the number of feet. Thus, in this situation, function notation may help avoid confusion.

If students are bothered by the formula, it can also be explained with the unit factors method used extensively in science classes, as indicated in the solution below.

Solution(s):

$P(x)$ **Mexican pesos** $= x$ **Japanese yen** $\left(\frac{\textbf{0.138616 Mexican pesos}}{\textbf{1 Japanese yen}}\right)$***; thus,***

$P(x) = 0.138616x$***, where x is a number of Japanese yen and P(x) is the corresponding number of Mexican pesos.***

b. Using the rates above, write a function *E* that converts from Mexican pesos to Euros.

Comment(s):
This part is similar to part a, but here students must determine the meaning of x and E(x).

Solution(s):
$E(x) = 0.0547265x$***, where x is a number of Mexican pesos and E(x) is the corresponding number of Euros.***