Probability of Independent and Dependent Events

CCM2 Unit 6: Probability

Independent and Dependent Events

- **Independent Events:** two events are said to be independent when one event has no affect on the probability of the other event occurring.
- **Dependent Events:** two events are dependent if the outcome or probability of the first event affects the outcome or probability of the second.

Independent Events

Suppose a die is rolled and then a coin is tossed.

- Explain why these events are independent.
  - They are independent because the outcome of rolling a die does not affect the outcome of tossing a coin, and vice versa.
- We can construct a table to describe the sample space and probabilities:

	Roll 1	Roll 2	Roll 3	Roll 4	Roll 5	Roll 6
Head						
Tail						

	Roll 1	Roll 2	Roll 3	Roll 4	Roll 5	Roll 6
Head	1,H	2,H	3,H	4,H	5,H	6,H
Tail	1,T	2,T	3,T	4,T	5,T	6,T

- How many outcomes are there for rolling the die?
  - 6 outcomes
- How many outcomes are there for tossing the coin?
  - 2 outcomes
- How many outcomes are there in the sample space of rolling the die and tossing the coin?
  - 12 outcomes

	Roll 1	Roll 2	Roll 3	Roll 4	Roll 5	Roll 6
Head	1,H	2,H	3,H	4,H	5,H	6,H
Tail	1,T	2,T	3,T	4,T	5,T	6,T

- Is there another way to decide how many outcomes are in the sample space?
    - Multiply the number of outcomes in each event together to get the total number of outcomes.
- Let’s see if this works for another situation.

A fast food restaurant offers 5 sandwiches and 3 sides. How many different meals of a sandwich and side can you order?

- If our theory holds true, how could we find the number of outcomes in the sample space?
    - 5 sandwiches x 3 sides = 15 meals
- Make a table to see if this is correct.

	Sand. 1	**Sand. 2**	**Sand. 3**	**Sand. 4**	**Sand. 5**
Side 1					
Side 2					
Side 3					

- Were we correct?

Probabilities of Independent Events

The probability of independent events is the probability of both occurring, denoted by P(A and B) or $P(A \cap B)$.

	Roll 1	Roll 2	Roll 3	Roll 4	Roll 5	Roll 6
Head	1,H	2,H	3,H	4,H	5,H	6,H
Tail	1,T	2,T	3,T	4,T	5,T	6,T

Use the table to find the following probabilities:

1. P(rolling a 3)

   2/12 = 1/6

2. P(Tails)

   6/12 = ½

3. P(rolling a 3 AND getting tails)

   1/12

4. P(rolling an even)

   6/12 = ½

5. P(heads)

   6/12 = ½

6. P(rolling an even AND getting heads)

   3/12 or 1/4

What do you notice about the answers to 3 and 6?

Multiplication Rule of Probability

- The probability of two independent events occurring can be found by the following formula:

$$P(A \cap B) = P(A) \times P(B)$$

Examples

1. At City High School, 30% of students have part-time jobs and 25% of students are on the honor roll. What is the probability that a student chosen at random has a part-time job and is on the honor roll? Write your answer in context.

    P(PT job and honor roll) = P(PT job) x P(honor roll) = .30 x .25 = .075

    There is a 7.5% probability that a student chosen at random will have a part-time job and be on the honor roll.

2. The following table represents data collected from a grade 12 class in DEF High School.

Plans after High School

Gender	University	Community College	Total
Males	28	56	84
Females	43	37	80
Total	71	93	164

Suppose 1 student was chosen at **random** from the grade 12 class.

(a) What is the probability that the student is female?

(b) What is the probability that the student is going to university?

Now suppose 2 people both randomly chose 1 student from the grade 12 class. Assume that it's possible for them to choose the same student.

(c) What is the probability that the first person chooses a student who is female and the second person chooses a student who is going to university?

3. Suppose a card is chosen at random from a deck of cards, replaced, and then a second card is chosen.

- Would these events be independent? How do we know?
  - Yes, because the first card is replaced before the second card is drawn.
- What is the probability that both cards are 7s?
  - P(7) = 4/52, so P(7 and 7) = P(7) x P(7) = 4/52 x 4/52 = 1/169 or .0059.
  - This means that the probability of drawing a 7, replacing the card and then drawing another 7 is 0.59%

Dependent Events

- Remember, we said earlier that
    - **Dependent Events:** two events are dependent if the outcome or probability of the first event affects the outcome or probability of the second.
- Let's look at some scenarios and determine whether the events are independent or dependent.

Determine whether the events are independent or dependent:

1. Selecting a marble from a container and selecting a jack from a deck of cards.
    - Independent
2. Rolling a number less than 4 on a die and rolling a number that is even on a second die.
    - Independent
3. Choosing a jack from a deck of cards and choosing another jack, without replacement.
    - Dependent
4. Winning a hockey game and scoring a goal.
    - Dependent

Probabilities of Dependent Events

- We cannot use the multiplication rule for finding probabilities of dependent events because the one event affects the probability of the other event occurring.
- Instead, we need to think about how the occurrence of one event will effect the sample space of the second event to determine the probability of the second event occurring.
- Then we can multiply the new probabilities.

Examples

1. Suppose a card is chosen at random from a deck, the card is NOT replaced, and then a second card is chosen from the same deck. What is the probability that both will be 7s?
    - This is similar the earlier example, but these events are dependent? How do we know?
    - How does the first event affect the sample space of the second event?

1. Suppose a card is chosen at random from a deck, the card is NOT replaced, and then a second card is chosen from the same deck. What is the probability that both will be 7s?

- Let’s break down what is going on in this problem:
  - We want the probability that the first card is a 7, or P(1st is 7), and the probability that the second card is a 7, or P(2nd is 7).
  - P(1st is 7) = 4/52 because there a four 7s and 52 cards
  - How is P(2nd is 7) changed by the first card being a 7?
  - P(2nd is 7) = 3/51
  - P(1st is 7, 2nd is 7) = 4/52 x 3/51 = 1/221 or .0045
  - The probability of drawing two sevens without replacement is 0.45%

2. A box contains 5 red marbles and 5 purple marbles. What is the probability of drawing 2 purple marbles and 1 red marble in succession *without replacement*?

- P(1^{st} purple) = 5/10
- P(2^{nd} purple) = 4/9
- P(3^{rd} red) = 5/8
- P(purple,purple,red) = 5/10 x 4/9 x 5/8 = 5/36 or .139
- The probability of drawing a purple, a purple, then a red without replacement is 13.9%

3. In Example 2, what is the probability of first drawing all 5 red marbles in succession and then drawing all 5 purple marbles in succession *without replacement*?

- P(5 red then 5 purple) = (5/10)(4/9)(3/8)(2/7)(1/6)(5/5)(4/4)(3/3)(2/2)(1/1) = 1/252 or .004
- The probability of drawing 5 red then 5 purple without replacement is 0.4%
- Explain why the last 5 probabilities above were all equivalent to 1.
- This is because there were only purple marbles left, so the probability for drawing a purple marble was 1.