

# NC Math 2

## Unit #6 Probability



"I wish we hadn't learned probability 'cause I don't think our odds are good."



## Guided Notes: Sample Spaces, Subsets, and Basic Probability

### Sample Space:

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List the sample space,  $S$ , for each of the following:

- Tossing a coin:
- Rolling a six-sided die:
- Drawing a marble from a bag that contains two red, three blue, and one white marble:

### Union of two sets ( $A \cup B$ ):

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### Intersection of two sets ( $A \cap B$ ):

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Example: Given the following sets, find  $A \cap B$  and  $A \cup B$

$$A = \{1,3,5,7,9,11,13,15\} \quad B = \{0,3,6,9,12,15\}$$

$$A \cap B = \underline{\hspace{2cm}} \quad A \cup B = \underline{\hspace{2cm}}$$

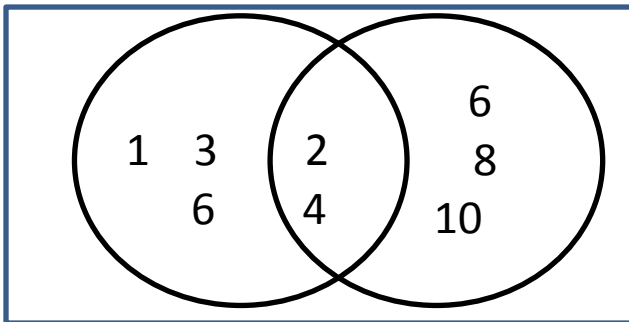
### Venn Diagram:

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Picture:

Example: Use the Venn Diagram to answer the following questions:



- What are the elements of set A?
- What are the elements of set B?
- Why are 1, 2, and 4 in both sets?
- What is  $A \cap B$ ?
- What is  $A \cup B$ ?

## Compliment of a set:

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• Ex:  $S = \{\dots-3,-2,-1,0,1,2,3,4,\dots\}$

$A = \{\dots-2,0,2,4,\dots\}$

If A is a subset of S, what is  $A^c$ ? \_\_\_\_\_

Example: Use the Venn Diagram above to find the following:

9. What is  $A^c$ ? \_\_\_\_\_  $B^c$ ? \_\_\_\_\_

10. What is  $(A \cap B)^c$ ? \_\_\_\_\_

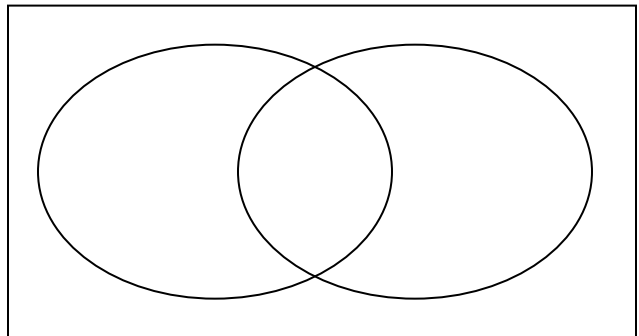
11. What is  $(A \cup B)^c$ ? \_\_\_\_\_

Example: In a class of 60 students, 21 sign up for chorus (A), 29 sign up for band (B), and 5 take both. 15 students in the class are not enrolled in either band or chorus.

6. Put this information into a Venn Diagram. If the sample space, S, is the set of all students in the class, let students in chorus be set A and students in band be set B.

7. What is  $A \cup B$ ? \_\_\_\_\_

8. What is  $A \cap B$ ? \_\_\_\_\_



A survey of used car salesmen revealed the following information:

24 wear white patent-leather shoes

28 wear plaid trousers

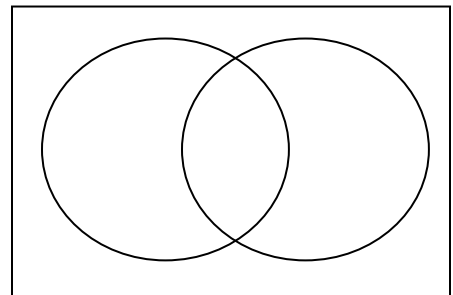
20 wear both of these things

2 wear neither of these things

Create a Venn diagram.

How many salesmen wore plaid trousers or white shoes but not both?

How many salesmen didn't wear trousers?



## Two way Tables

How many females are there?

How many left handed males are there?

How many right handed people are there?

Gender compared to handedness of Year 8 students

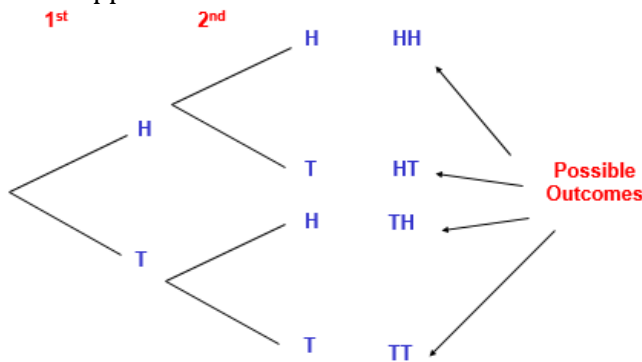
	Handed		
	Left	Right	
Female	7	46	53
Male	5	63	68
	12	109	121

## Tree Diagrams

A way of showing the possibilities of two or more events

Helps to calculate the probabilities of events occurring

Example of a fair coin flipped twice



Let's make a tree diagram to show the different combinations you could make if you could select between white and chocolate milk and either chocolate chip or oatmeal cookies.

## Basic Probability

Probability of an Event:  $P(E) = \underline{\hspace{2cm}}$

Note that  $P(A^c)$  is every outcome **except (or not)** A, so we can find  $P(A^c)$  by finding  $\underline{\hspace{2cm}}$ .

Why do you think this works?  $\underline{\hspace{2cm}}$

Example: An experiment consists of tossing three coins.

12. List the sample space for the outcomes of the experiment.

$\underline{\hspace{10cm}}$

13. Find the following probabilities:

P(all heads)  $\underline{\hspace{2cm}}$       P(two tails)  $\underline{\hspace{2cm}}$       P(no heads)  $\underline{\hspace{2cm}}$

P(at least one tail)  $\underline{\hspace{2cm}}$

How could you use compliments to find d?

$\underline{\hspace{10cm}}$

Example: A bag contains six red marbles, four blue marbles, two yellow marbles and 3 white marbles. One marble is drawn at random.

14. List the sample space for this experiment. \_\_\_\_\_

15. Find the following probabilities:

P(red) \_\_\_\_\_ P(blue or white) \_\_\_\_\_ P(not yellow) \_\_\_\_\_

Note that we could either count all the outcomes that are not yellow or we could think of this as being  $1 - P(\text{yellow})$ . Why is this?

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Example: A card is drawn at random from a standard deck of cards. Find each of the following:

16. P(heart) \_\_\_\_\_

17. P(black card) \_\_\_\_\_

18. P(2 or jack) \_\_\_\_\_

19. P(not a heart) \_\_\_\_\_

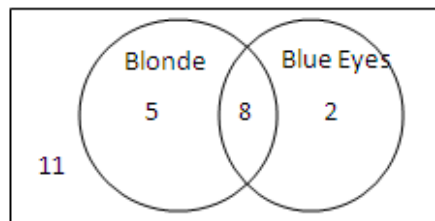
Example:

P(blonde hair)

P(blonde hair and blue eyes)

P (blonde hair or blue eyes)

P (not blue eyes)



**Odds:** The odds of an event occurring are equal to the ratio of \_\_\_\_\_ to \_\_\_\_\_.

Odds = \_\_\_\_\_

17. The weather forecast for Saturday says there is a 75% chance of rain. What are the odds that it will rain on Saturday?

- What does the 75% in this problem mean?
- The favorable outcome in this problem is that it rains:
- Odds(rain) =
- Should you make outdoor plans for Saturday?

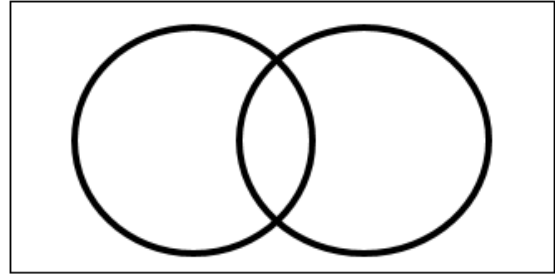
18. What are the odds of drawing an ace at random from a standard deck of cards?

## Intro to Probability Homework

Organize the data into the circles.

Factors of 64: 1, 2, 4, 8, 16, 32, 64

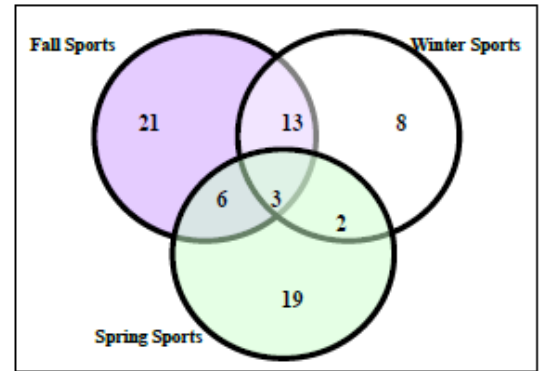
Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24



Answer Questions about the diagram below

How many students play sports year-round?

- 1) How many students play sports in the spring and fall?
- 2) How many students play sports in the winter and fall?
- 3) How many students play sports in the winter and spring?
- 4) How many students play only one sport?
- 5) How many students play at least two sports?
6. Suppose you have a standard deck of 52 cards. Let:



A: draw a 7    B: draw a Diamond

a. Describe  $A \cup B$  for this experiment, and find the probability of  $A \cup B$ .

b. Describe  $A \cap B$  for this experiment, and find the probability of  $A \cap B$ .

7) Suppose a box contains three balls, one red, one blue, and one white. One ball is selected, its color is observed, and then the ball is placed back in the box. The balls are scrambled, and again, a ball is selected and its color is observed. What is the sample space of the experiment?

8) Suppose you have a jar of candies: 4 red, 5 purple and 7 green. Find the following probabilities of the following events:

Selecting a red candy. \_\_\_\_\_      Selecting a purple candy. \_\_\_\_\_

Selecting a green or red candy \_\_\_\_\_      Selecting a yellow candy. \_\_\_\_\_

Selecting any color except a green candy \_\_\_\_\_      Find the odds of selecting a red candy \_\_\_\_\_

Find the odds of selecting a purple or green candy \_\_\_\_\_

9) What is the sample space for a single spin of a spinner with red, blue, yellow and green sections spinner?

What is the sample space for 2 spins of the first spinner?

If the spinner is equally likely to land on each color, what is the probability of landing on red in one spin?

What is the probability of landing on a primary color in one spin?

What is the probability of landing on green both times in two spins?

10) Consider the throw of a die experiment. Assume we define the following events:

A: Even number      B; number less than or equal to 3

Describe  $A \cup B$  for this experiment.

Describe  $A \cap B$  for this experiment.

Calculate  $P(A \cup B)$  and  $P(A \cap B)$ , assuming the die is fair.

Andy has asked his girlfriend to make all the decisions for their date on her birthday. She will pick a restaurant and an activity for the date. Andy will choose a gift for her. The local restaurants include Mexican, Chinese, Seafood, and Italian. The activities she can choose from are Putt-Putt, bowling, and movies. Andy will buy her either candy or flowers.

11. How many outcomes are there for these three decisions?

12. Draw a tree diagram to illustrate the choices.

A travel agent plans trips for tourists from Chicago to Miami. He gives them three ways to get from town to town: airplane, bus, train. Once the tourists arrive, there are two ways to get to the hotel: hotel van or taxi. The cost of each type of transportation is given in the table below.

Transportation Type	Cost
Airplane	\$350
Bus	\$150
Train	\$225
Hotel Van	\$60
Taxi	\$40

13. Draw a tree diagram to illustrate the possible choices for the tourists. Determine the cost for each outcome.

14. If the tourists were flying to New York, the subway would be a third way to get to the hotel. How would this change the number of outcomes? Use the Fundamental Counting Principle to explain your answer.



## Guided Notes: Mutually Exclusive and Inclusive events

### Mutually Exclusive Events

Suppose you are rolling a six-sided die. What is the probability that you roll an odd number or you roll a 2?

- Can these both occur at the same time? Why or why not?
- 

### Mutually Exclusive (Disjoint) Events:

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- The probability of two mutually exclusive events occurring at the same time,  $P(A \text{ and } B)$ , is \_\_\_\_\_

To find the probability of one of two **mutually exclusive** events occurring, use the following formula:

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### EASY WAY TO REMEMBER:

### Examples:

1. If you randomly chose one of the integers 1 - 10, what is the probability of choosing either an odd number or an even number?

Are these mutually exclusive events? Why or why not? \_\_\_\_\_

Complete the following statement:

$$P(\text{odd or even}) = P(\text{____}) + P(\text{____})$$

Now fill in with numbers:

$$P(\text{odd or even}) = \text{____} + \text{____} = \text{_____}$$

Does this answer make sense? \_\_\_\_\_

2. Two fair dice are rolled. What is the probability of getting a sum less than 7 or a sum equal to 10? Are these events mutually exclusive? \_\_\_\_\_

Sometimes using a table of outcomes is useful. Complete the following table using the sums of two dice:

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

P(getting a sum less than 7 OR sum of 10) = \_\_\_\_\_

This means \_\_\_\_\_

### Mutually Inclusive Events

Suppose you are rolling a six-sided die. What is the probability that you roll an odd number or a number less than 4?

- Can these both occur at the same time? If so, when?

\_\_\_\_\_

\_\_\_\_\_

### Mutually Inclusive Events:

\_\_\_\_\_

\_\_\_\_\_

### Probability of the Union of Two Events: The Addition Rule:

\_\_\_\_\_

\*\*\* \_\_\_\_\_ \*\*\*

### Examples:

1. What is the probability of choosing a card from a deck of cards that is a club or a ten?  
P(choosing a club or a ten) =
  
2. What is the probability of choosing a number from 1 to 10 that is less than 5 or odd?

3. A bag contains 26 tiles with a letter on each, one tile for each letter of the alphabet. What is the probability of reaching into the bag and randomly choosing a tile with one of the first 10 letters of the alphabet on it or randomly choosing a tile with a vowel on it?
4. A bag contains 26 tiles with a letter on each, one tile for each letter of the alphabet. What is the probability of reaching into the bag and randomly choosing a tile with one of the last 5 letters of the alphabet on it or randomly choosing a tile with a vowel on it?
5.  $P(A) = 0.7$ ,  $P(B) = 0.4$ ,  $P(A \text{ and } B) = 0.3$
- a) Make a Venn Diagram representing the situation.
  - b) Are A and B mutually exclusive?
  - c) Are A and B independent events?
  - d) Find  $P(A \text{ or } B)$
  - e.) EXTENSION: Find the following probabilities:  
 $P(A)^c$            $P(A \text{ and } B)^c$            $P(A \text{ or } B)^c$

## Mutually Exclusive and Inclusive Events Homework

1. 2 dice are tossed. What is the probability of obtaining a sum equal to 6?
2. 2 dice are tossed. What is the probability of obtaining a sum less than 6?
3. 2 dice are tossed. What is the probability of obtaining a sum of at least 6?
4. Thomas bought a bag of jelly beans that contained 10 red jelly beans, 15 blue jelly beans, and 12 green jelly beans. What is the probability of Thomas reaching into the bag and pulling out a blue or green jelly bean?
5. A card is chosen at random from a standard deck of cards. What is the probability that the card chosen is a heart or spade? Are these events mutually exclusive?
6. 3 coins are tossed simultaneously. What is the probability of getting 3 heads or 3 tails? Are these events mutually exclusive?
7. In question 6, what is the probability of getting 3 heads *and* 3 tails when tossing the 3 coins simultaneously?
8. Are randomly choosing a person who is left-handed and randomly choosing a person who is right-handed mutually exclusive events? Explain your answer.
9. Suppose 2 events are mutually exclusive events. If one of the events is randomly choosing a boy from the freshman class of a high school, what could the other event be? Explain your answer.
10. Consider a sample set as  $S = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$ . Event  $A$  is the multiples of 4, while event  $B$  is the multiples of 5. What is the probability that a number chosen at random will be from both  $A$  and  $B$ ?
11. For question 10, what is the probability that a number chosen at random will be from either  $A$  or  $B$ ?

12. A card is chosen at random from a standard deck of cards. What is the probability that the card chosen is a heart or a face card? Are these events mutually inclusive?

13. What is the probability of choosing a number from 1 to 10 that is greater than 5 or even?

14. Are randomly choosing a teacher and randomly choosing a father mutually exclusive events? Explain your answer.

15. Jack is a student in Bluenose High School. He noticed that a lot of the students in his math class were also in his chemistry class. In fact, of the 60 students in his grade, 28 students were in his math class, 32 students were in his chemistry class, and 15 students were in both his math class and his chemistry class. He decided to calculate what the probability was of selecting a student at random who was either in his math class or his chemistry class, but not both. Draw a Venn diagram and help Jack with his calculation.

- a. Are these events mutually exclusive?
- b. Find  $P(\text{Math or Chemistry})$
- c. Find  $P(\text{Math and Chemistry})$
- d. What is the difference between b. and c.?

16. Brenda did a survey of the students in her classes about whether they liked to get a candy bar or a new math pencil as their reward for positive behavior. She asked all 71 students she taught, and 32 said they would like a candy bar, 25 said they wanted a new pencil, and 4 said they wanted both. If Brenda were to select a student at random from her classes, what is the probability that the student chosen would want:

- a. a candy bar or a pencil?
- b. neither a candy bar nor a pencil?

	<b>Pencil</b>	<b>No Pencil</b>	<b>Total</b>
<b>Candy Bar</b>			
<b>No Candy Bar</b>			
<b>Total</b>			

## Guided Notes: Probability of Independent and Dependent Events

### Compound Events:

#### Independent Events:

#### Dependent Events:

Suppose a die is rolled and then a coin is tossed.

- Explain why these events are independent.

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Fill in the table to describe the sample space:

	Roll 1	Roll 2	Roll 3	Roll 4	Roll 5	Roll 6
Head						
Tail						

- How many outcomes are there for rolling the die? \_\_\_\_\_
- How many outcomes are there for tossing the coin? \_\_\_\_\_
- How many outcomes are there in the sample space of rolling the die and tossing the coin?  
\_\_\_\_\_
- Is there another way to decide how many outcomes are in the sample space?  
\_\_\_\_\_

Let's see if this works for another situation.

A fast food restaurant offers 5 sandwiches and 3 sides. How many different meals of a sandwich and side can you order?

- If our theory holds true, how could we find the number of outcomes in the sample space?  
\_\_\_\_\_

- Make a table to see if this is correct.


- Were we correct? \_\_\_\_\_

### Probabilities of Independent Events

The probability of independent events is \_\_\_\_\_, denoted by \_\_\_\_\_.

	Roll 1	Roll 2	Roll 3	Roll 4	Roll 5	Roll 6
Head						
Tail						

Fill in the table again and then use the table to find the following probabilities:

1.  $P(\text{rolling a 3}) =$  \_\_\_\_\_
2.  $P(\text{Tails}) =$  \_\_\_\_\_
3.  $P(\text{rolling a 3 AND getting tails}) =$  \_\_\_\_\_
4.  $P(\text{rolling an even}) =$  \_\_\_\_\_
5.  $P(\text{heads}) =$  \_\_\_\_\_
6.  $P(\text{rolling an even AND getting heads}) =$  \_\_\_\_\_

What do you notice about the answers to 3 and 6?

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## Multiplication Rule of Probability

- The probability of two independent events occurring can be found by the following formula:
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### Examples:

- At City High School, 30% of students have part-time jobs and 25% of students are on the honor roll. What is the probability that a student chosen at random has a part-time job and is on the honor roll? Write your answer in context.
- The following table represents data collected from a grade 12 class in DEF High School.

Plans after High School			
Gender	University	Community College	Total
Males	28	56	84
Females	43	37	80
Total	71	93	164

Suppose 1 student was chosen at **random** from the grade 12 class.

(a) What is the probability that the student is female? \_\_\_\_\_

(b) What is the probability that the student is going to university? \_\_\_\_\_

Now suppose 2 people both randomly chose 1 student from the grade 12 class. Assume that it's possible for them to choose the same student.

(c) What is the probability that the first person chooses a student who is female and the second person chooses a student who is going to university?

\_\_\_\_\_

- Suppose a card is chosen at random from a deck of cards, replaced, and then a second card is chosen.

Would these events be independent? How do we know?

\_\_\_\_\_

What is the probability that both cards are 7s?

\_\_\_\_\_



## Probabilities of Dependent Events

Determine whether the events are independent or dependent:

1. Selecting a marble from a container and selecting a jack from a deck of cards. \_\_\_\_\_
2. Rolling a number less than 4 on a die and rolling a number that is even on a second die.  
\_\_\_\_\_
3. Choosing a jack from a deck of cards and choosing another jack, without replacement.  
\_\_\_\_\_
4. Winning a hockey game and scoring a goal. \_\_\_\_\_

- We cannot use the multiplication rule for finding probabilities of dependent events because the one event affects the probability of the other event occurring.
- Instead, we need to think about how the occurrence of one event will effect the sample space of the second event to determine the probability of the second event occurring.
- Then we can multiply the new probabilities.

### Examples:

1. Suppose a card is chosen at random from a deck, the card is NOT replaced, and then a second card is chosen from the same deck. What is the probability that both will be 7s?
  - This is similar the earlier example, but these events are dependent? How do we know? \_\_\_\_\_
  - How does the first event affect the sample space of the second event?  
\_\_\_\_\_

Now find the probability that both cards will be 7s.

2. A box contains 5 red marbles and 5 purple marbles. What is the probability of drawing 2 purple marbles and 1 red marble in succession *without replacement*?
3. In Example 2, what is the probability of first drawing all 5 red marbles in succession and then drawing all 5 purple marbles in succession *without replacement*?

## Independent and Dependent Events Homework

- Determine which of the following are examples of independent or dependent events.
  - Rolling a 5 on one die and rolling a 5 on a second die.
  - Choosing a cookie from the cookie jar and choosing a jack from a deck of cards.
  - Selecting a book from the library and selecting a book that is a mystery novel.
  - Going to the beach and bringing an umbrella.
  - Getting gasoline for your car and getting diesel fuel for your car.
  - Choosing an 8 from a deck of cards, replacing it, and choosing a face card.
  - Choosing a jack from a deck of cards and choosing another jack, without replacement.
  - Being lunchtime and eating a sandwich.
- A coin and a die are tossed. Calculate the probability of getting tails and a 5.
- In Tania's homeroom class, 9% of the students were born in March and 40% of the students have a blood type of O+. What is the probability of a student chosen at random from Tania's homeroom class being born in March and having a blood type of O+?
- If a baseball player gets a hit in 31% of his at-bats, what is the probability that the baseball player will get a hit in 5 at-bats in a row?
- What is the probability of tossing 2 coins one after the other and getting 1 head and 1 tail?
- 2 cards are chosen from a deck of cards. The first card is replaced before choosing the second card. What is the probability that they both will be clubs?
- 2 cards are chosen from a deck of cards. The first card is replaced before choosing the second card. What is the probability that they both will be face cards?
- If the probability of receiving at least 1 piece of mail on any particular day is 22%, what is the probability of *not* receiving any mail for 3 days in a row?

9. Johnathan is rolling 2 dice and needs to roll an 11 to win the game he is playing. What is the probability that Johnathan wins the game?
10. Thomas bought a bag of jelly beans that contained 10 red jelly beans, 15 blue jelly beans, and 12 green jelly beans. What is the probability of Thomas reaching into the bag and pulling out a blue or green jelly bean and then reaching in again and pulling out a red jelly bean? Assume that the first jelly bean is not replaced.
11. For question 10, what if the order was reversed? In other words, what is the probability of Thomas reaching into the bag and pulling out a red jelly bean and then reaching in again and pulling out a blue or green jelly bean *without replacement*?
12. What is the probability of drawing 2 face cards one after the other from a standard deck of cards *without replacement*?
13. There are 3 quarters, 7 dimes, 13 nickels, and 27 pennies in Jonah's piggy bank. If Jonah chooses 2 of the coins at random one after the other, what is the probability that the first coin chosen is a nickel and the second coin chosen is a quarter? Assume that the first coin is not replaced.
14. For question 13, what is the probability that neither of the 2 coins that Jonah chooses are dimes? Assume that the first coin is not replaced.
15. Jenny bought a half-dozen doughnuts, and she plans to randomly select 1 doughnut each morning and eat it for breakfast until all the doughnuts are gone. If there are 3 glazed, 1 jelly, and 2 plain doughnuts, what is the probability that the last doughnut Jenny eats is a jelly doughnut?
16. Steve will draw 2 cards one after the other from a standard deck of cards *without replacement*. What is the probability that his 2 cards will consist of a heart and a diamond?

## Guided Notes: Conditional Probability

### Conditional Probability: -

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Examples of conditional probability:

The conditional probability of A given B is expressed as \_\_\_\_\_

The formula is: \_\_\_\_\_

Examples of Conditional Probability:

1. You are playing a game of cards where the winner is determined by drawing two cards of the same suit. What is the probability of drawing clubs on the second draw if the first card drawn is a club?
  
  
  
  
  
  
  
  
  
  
2. A bag contains 6 blue marbles and 2 brown marbles. One marble is randomly drawn and discarded. Then a second marble is drawn. Find the probability that the second marble is brown given that the first marble drawn was blue.
  
  
  
  
  
  
  
  
  
  
3. In Mr. Jonas' homeroom, 70% of the students have brown hair, 25% have brown eyes, and 5% have both brown hair and brown eyes. A student is excused early to go to a doctor's appointment. If the student has brown hair, what is the probability that the student also has brown eyes?

## Using Two-Way Frequency Tables to Compute Conditional Probabilities

1. Suppose we survey all the students at school and ask them how they get to school and also what grade they are in. The chart below gives the results. Complete the two-way frequency table:

	Bus	Walk	Car	Other	Total
9 <sup>th</sup> or 10 <sup>th</sup>	106	30	70	4	
11 <sup>th</sup> or 12 <sup>th</sup>	41	58	184	7	
Total					

Suppose we randomly select one student.

- What is the probability that the student walked to school?
  - $P(9^{\text{th}} \text{ or } 10^{\text{th}} \text{ grader})$
  - $P(\text{rode the bus OR } 11^{\text{th}} \text{ or } 12^{\text{th}} \text{ grader})$
  - What is the probability that a student is in 11th or 12th grade *given that* they rode in a car to school?
  - What is  $P(\text{Walk}|9^{\text{th}} \text{ or } 10^{\text{th}} \text{ grade})$ ?
2. The manager of an ice cream shop is curious as to which customers are buying certain flavors of ice cream. He decides to track whether the customer is an adult or a child and whether they order vanilla ice cream or chocolate ice cream. He finds that of his 224 customers in one week that 146 ordered chocolate. He also finds that 52 of his 93 adult customers ordered vanilla. Build a two-way frequency table that tracks the type of customer and type of ice cream.

	Vanilla	Chocolate	Total
Adult			
Child			
Total			

- Find  $P(\text{vanilla}|\text{adult})$
  - Find  $P(\text{child}|\text{chocolate})$
3. A survey asked students which types of music they listen to? Out of 200 students, 75 indicated pop music and 45 indicated country music with 22 of these students indicating they listened to both. Use a Venn diagram to find the probability that a randomly selected student listens to pop music given that they listen country music.

## Conditional Probability Homework

1. Complete the following table using sums from rolling two dice. Use the table to answer questions 2-5.

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

2. 2 fair dice are rolled. What is the probability that the sum is even given that the first die that is rolled is a 2?
3. 2 fair dice are rolled. What is the probability that the sum is even given that the first die rolled is a 5?
4. 2 fair dice are rolled. What is the probability that the sum is odd given that the first die rolled is a 5?
5. Steve and Scott are playing a game of cards with a standard deck of playing cards. Steve deals Scott a black king. What is the probability that Scott's second card will be a red card?
6. Sandra and Karen are playing a game of cards with a standard deck of playing cards. Sandra deals Karen a red seven. What is the probability that Karen's second card will be a black card?
7. Donna discusses with her parents the idea that she should get an allowance. She says that in her class, 55% of her classmates receive an allowance for doing chores, and 25% get an allowance for doing chores and are good to their parents. Her mom asks Donna what the probability is that a classmate will be good to his or her parents given that he or she receives an allowance for doing chores. What should Donna's answer be?
8. At a local high school, the probability that a student speaks English and French is 15%. The probability that a student speaks French is 45%. What is the probability that a student speaks English, given that the student speaks French?
9. On a game show, there are 16 questions: 8 easy, 5 medium-hard, and 3 hard. If contestants are given questions randomly, what is the probability that the first two contestants will get easy questions?
10. On the game show above, what is the probability that the first contestant will get an easy question and the second contestant will get a hard question?

11. Figure 2.2 shows the counts of earned degrees for several colleges on the East Coast. The level of degree and the gender of the degree recipient were tracked. Row & Column totals are included.

- What is the probability that a randomly selected degree recipient is a female?
- What is the probability that a randomly chosen degree recipient is a man?
- What is the probability that a randomly selected degree recipient is a woman, given that they received a Master's Degree?
- For a randomly selected degree recipient, what is  $P(\text{Bachelor's Degree}|\text{Male})$ ?

	Bachelor's	Master's	Professional	Doctorate	Total
Female	542	128	26	18	714
Male	438	165	38	20	661
Total	980	293	64	38	1375

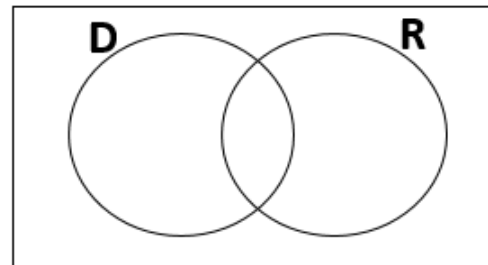
12. Animals on the endangered species list are given in the table below by type of animal and whether it is domestic or foreign to the United States. Complete the table and answer the following questions.

	Mammals	Birds	Reptiles	Amphibians	Total
United States	63	78	14	10	
Foreign	251	175	64	8	
Total					

An endangered animal is selected at random. What is the probability that it is:

- a bird found in the United States?
- foreign or a mammal?
- domestic?
- a bird given that it is found in the United States?
- a bird given that it is foreign?

13. Using the table in #10, fill in the Venn Diagram looking at the events **D**: Domestic and **R**: Reptiles.



Answer the questions below.

- $P(D \text{ and } R)$
- $P(D \text{ or } R)$
- $P(R^c)$
- $P(D | R)$
- $P(R | D)$
- Write in words what part d.) and part e.) notation means and although they appear similar, why are their probabilities different?
- Challenge! Explain what  $P(R^c | D)$  is asking for and then find:

14. Every morning I buy either the Times or The Mail. The probability that I buy the Times is  $\frac{3}{4}$  and the probability that I buy the Mail is  $\frac{1}{4}$ . If I buy the Times, the probability that I complete the crossword is  $\frac{2}{5}$ , whereas if I buy the Mail the probability that I complete the crossword is  $\frac{4}{5}$ .

a. Draw a tree diagram with the probability to represent this situation.

b. In fraction form, find  $P(\text{do not complete the crossword} \mid \text{I buy The Mail})$ .

c. Find the probability that I complete the crossword on any particular day.

d. *Challenge!* If I completed the crossword, find the probability that I bought The Mail.

15. A group of 64 people were surveyed about the type of movies they prefer. 12 females preferred romantic comedies, 10 females preferred action movies, and 3 females preferred horror movies. 8 males preferred romantic comedies, 25 males preferred action movies, and 6 males preferred horror movies.

a. Construct a two-way frequency table to organize this data.

b. Let  $F$  be the event that the person is female. Find  $P(F)$ .

c. Let  $R$  be the event that the person prefers romantic comedies. Find  $P(R)$ .

d. Find  $P(F|R)$  and  $P(R|F)$ . Explain how these two calculations are different.

e. Are events  $F$  and  $R$  independent? Justify your answer.



Are they independent?

Let's review! To this point, we have decided whether events are independent just by thinking about whether there is an overlap between the two events. Try these:

**Which of these sets of events are independent?**

1. Tossing two dice and getting a 6 on both of them.
2. Choosing a marble out of a bag, then choosing another marble without replacing it.
3. Picking a King from a deck of cards and rolling a 4 on a die.
4. Using numbers 1 – 20, picking an odd number and picking a number less than 10.

Sometimes, there are events that are not quite as obvious.

For example, let's consider what customers order at lunch. The data is displayed in the chart below. Are the events "drinking water" and "ordering salad" independent?



There's definitely overlap here – there are 8 people who did both. But does that really mean that they're not independent?

**There are two ways to determine mathematically whether items are independent.**

The first way is to show that  $P(A) \cdot P(B) = P(A \cap B)$

	Orders salad	Orders sandwich
Drinks water	8	16
Drinks diet soda	6	12

	Orders salad	Orders sandwich	Total
Drinks water	8	16	
Drinks diet soda	6	12	
Total			42

In this situation, that means that the probability of drinking water times the probability of ordering a salad has to equal the probability of doing both things. So let's fill in the totals and see what we get.

Now let's find the probabilities:

$$P(\text{drinking water}) = \frac{1}{42} \qquad P(\text{ordering salad}) = \frac{1}{42}$$

$$P(\text{drinking water and ordering salad}) = \frac{1}{42}$$

Now fit the probabilities into the formula:

$$P(\text{drinking water}) \cdot P(\text{ordering salad}) = P(\text{drinking water and ordering salad})$$

$$\frac{1}{42} \cdot \frac{1}{42} = \frac{1}{42}$$

Use your calculator: \_\_\_\_\_ = \_\_\_\_\_

Check it out! Those two events really ARE independent!

There's another way to show independence. It uses the definition of independence – that the probability of event A will not change whether event B changes or not. If we translate that into an equation, we get this:

$$P(A) = P(A | B)$$

Let's look at the same example to see how this works:

	Orders salad	Orders sandwich	Total
Drinks water	8	16	24
Drinks diet soda	6	12	18
Total	14	28	42

In words, we have to show that the probability of ordering salad (which we already found!) is equal to the probability of ordering salad, given that the person is drinking water. So let's find the conditional probability:

$$P(\text{salad}|\text{water}) = \frac{1}{24}$$

Now let's set them equal:

$$\frac{1}{42} = \frac{1}{24}$$

Now do the math: \_\_\_\_\_ = \_\_\_\_\_

Wow! They're independent again! That shouldn't be much of a surprise – if two events come out to be independent one way, doesn't it make sense that they would be independent the other way, too? In fact, if you checked two other events from the table, they'll come out independent, too.

Let's try it! Let's figure out whether ordering diet soda and ordering a sandwich are independent. Do it both ways:

$$P(A) \cdot P(B) = P(A \cap B)$$

Put it in words first:

Then find the probabilities:

Then do the math:

Then write your conclusion in words:

$$P(A) = P(A | B)$$

Put it in words first:

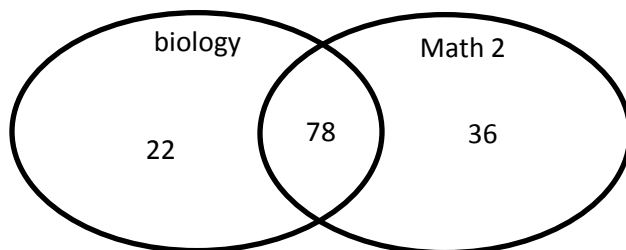
Then find the probabilities:

Then do the math:

Then write your conclusion in words:

You're probably thinking that one way is easier than the other and you're asking yourself "Why do I need two different ways? Why can't I just learn one way?" The problem is that sometimes one way is easier than the other, so it's good to know both.

Let's look at a Venn diagram of the number of students enrolled in both Math 2 and Biology during fall semester:



Are taking biology and taking Math 2 independent?

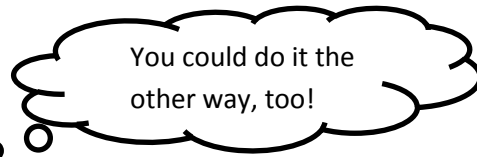
Let's use  $P(A) = P(A | B)$ .

In words:  $P(\text{biology}) = P(\text{biology} | \text{math 2})$

$$\frac{22+78}{136} = \frac{78}{22+78}$$

$$\frac{100}{136} = \frac{78}{100}$$

$$.7353 \neq .78$$



These two events are NOT independent.

You can also use these formulas to find probabilities. Here's an example:

**Events  $A$  and  $B$  are independent.**  
**If  $P(A) = 0.21$ ,  $P(B) = 0.34$ , find  $P(A \text{ and } B)$ .**

The key here is knowing that the events are independent. Since they're independent, we know that  $P(A) \cdot P(B) = P(A \cap B)$ . Just plug in what we know:

$$P(A) \cdot P(B) = P(A \cap B)$$

$$0.21(0.34) = P(A \cap B)$$

$$0.0714 = P(A \cap B)$$

Now you try one: Find  $P(A)$  if  $P(B) = 0.8$ ,  $P(A \text{ and } B) = 0.40$

## Practice

1. In Mr. Jonas' homeroom, 70% of the students have brown hair, 25% have green eyes, and 5% have both brown hair and green eyes. A student is excused early to go to a doctor's appointment. If the student has brown hair, what is the probability that the student also has green eyes? Let  $A$  = brown hair and  $B$  = green eyes. Are events  $A$  and  $B$  independent?

2. Determine whether age and choice of ice cream are independent events. (you can pick either age or either flavor as event A and event B)

	<b>Vanilla</b>	<b>Chocolate</b>	<b>Total</b>
Adult	52	41	93
Child	26	105	131
Total	78	146	224

3. A survey of consumers in Mudville showed 10% were dissatisfied with their plumbing jobs. Half of these complaints dealt with Mr. Badwrench, who does 40% of the plumbing jobs in Mudville. What is the probability that a randomly selected consumer has an unsatisfactory plumbing job, given that the plumber was Mr. Badwrench. Are bad plumbing jobs and Mr. Badwrench independent?

4. Are having lung cancer and smoking independent? (the numbers in these tables are probabilities – they work the same way!)

	<b>Smoker</b>	<b>Nonsmoker</b>	<b>Total</b>
Lung cancer	0.12	0.03	0.15
No lung cancer	0.04	0.81	0.85
Total	0.16	0.84	1

5. Carrie is a kicker on her rugby team. She estimates that her chances of scoring on a penalty kick during a game are 75% when there is no wind, but only 60% on a windy day. The weather forecast gives a 55% probability of windy weather today. Are the events “Carrie scores on a penalty kick” and “it’s windy” independent?

	<b>Makes the kick</b>	<b>Misses the kick</b>	<b>Total</b>
Windy			
Not windy			
Total			1

6. Are selecting a 2 and selecting a club from a standard deck independent?

## Investigation: Theoretical vs. Experimental Probability

### Part 1: Theoretical Probability

Probability is the chance or likelihood of an event occurring. We will study two types of probability, theoretical and experimental.

**Theoretical Probability:** the probability of an event is the ratio or the number of favorable outcomes to the total possible outcomes.

$$P(\text{Event}) = \frac{\text{Number of favorable outcomes}}{\text{Total possible outcomes}}$$

**Sample Space:** The set of all possible outcomes. For example, the sample space of tossing a coin is {Heads, Tails} because these are the only two possible outcomes. Theoretical probability is based on the set of all possible outcomes, or the sample space.

1. List the sample space for rolling a six-sided die (remember you are listing a set, so you should use brackets {}):

Find the following probabilities:

P(2)

P(3 or 6)

P(odd)

P(not a 4)

P(1,2,3,4,5, or 6)

P(8)

2. List the sample space for tossing two coins:

Find the following probabilities:

P(two heads)

P(one head and one tail)

P(head, then tail)

P(all tails)

P(no tails)

3. Complete the sample space for tossing two six-sided dice:

{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),

(2,1), (2,2), (2,\_\_), \_\_, \_\_, \_\_,

(3,1), \_\_, \_\_, \_\_, \_\_, \_\_,

\_\_, \_\_, \_\_, \_\_, \_\_, \_\_,

\_\_, \_\_, \_\_, \_\_, \_\_, \_\_,

\_\_, \_\_, \_\_, \_\_, \_\_, \_\_}

Find the following probabilities:

P(a 1 and a 4)

P(a 1, then a 4)

P(sum of 8)

P(sum of 12)

P(doubles)

P(sum of 15)

4. When would you expect the probability of an event occurring to be 1, or 100%? Describe an event whose probability of occurring is 1.
5. When would you expect the probability of an event occurring to be 0, or 0%? Describe an event whose probability of occurring is 0.

## Part 2: Experimental Probability

**Experimental Probability:** the ratio of the number of times the event occurs to the total number of trials.

$$P(\text{Event}) = \frac{\text{Number or times the event occurs}}{\text{Total number of trials}}$$

- Do you think that theoretical and experimental probabilities will be the same for a certain event occurring? Explain your answer.
- Roll a six-sided die and record the number on the die. Repeat this 9 more times

Number on Die	Tally	Frequency
1		
2		
3		
4		
5		
6		
<b>Total</b>		10

Based on your data, find the following experimental probabilities:

$$P(2) \qquad P(3 \text{ or } 6) \qquad P(\text{odd}) \qquad P(\text{not a } 4)$$

How do these compare to the theoretical probabilities in Part 1? Why do you think they are the same or different?

- Record your data on the board (number on die and frequency only). Compare your data with other groups in your class. Explain what you observe about your data compared to the other groups. Try to make at least two observations.
- Combine the frequencies of all the groups in your class with your data and complete the following table:

Number on Die	Frequency
1	
2	
3	
4	
5	
6	
<b>Total</b>	

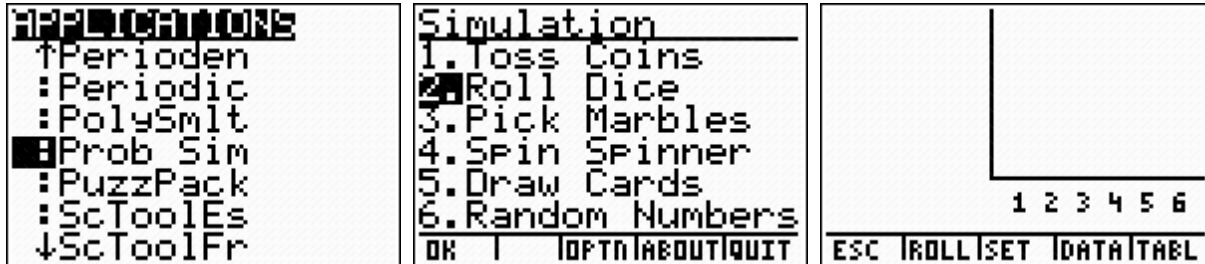
Based on the whole class data, find the following experimental probabilities:

$$P(2) \qquad P(3 \text{ or } 6) \qquad P(\text{odd}) \qquad P(\text{not a } 4)$$

How do these compare to your group's probabilities? How do these compare to the theoretical probabilities from Part 1?

What do you think would happen to the experimental probabilities if there were 200 trials? 500 trials? 1000 trials? 1,000,000 trials?

5. On your graphing calculator, go to APPS and open Prob Sim. Press any key and then select 2: Roll dice.



Click Roll. Notice that there will be a bar on the graph at the right. What does this represent?

Now push +1 nine more times. Push the right arrow to see the frequency of each number on the die. How many times did you get a 1? \_\_\_\_ A 2? \_\_\_\_ A 5?

Now press the +1, +10, and +50 buttons until you have rolled 100 times. Based on the data, find the following experimental probabilities:

P(2)                      P(3 or 6)                      P(odd)                      P(not a 4)

Press the +50 button until you have rolled 1000 times. Based on the data, find the following experimental probabilities:

P(2)                      P(3 or 6)                      P(odd)                      P(not a 4)

Press the +50 button until you have rolled 5000 times. Based on the data, find the following experimental probabilities:

P(2)                      P(3 or 6)                      P(odd)                      P(not a 4)

What can you expect to happen to the experimental probabilities in the long run? In other words, as the number of trials increases, what happens to the experimental probabilities?

Why can there be differences between experimental and theoretical probabilities in general?



### Part 3: Which one do I use?

So when do we use theoretical probability or experimental probability? Theoretical probability is always the best choice, when it can be calculated. But sometimes it is not possible to calculate theoretical probabilities because we cannot possibly know all of the possible outcomes. In these cases, experimental probability is appropriate. For example, if we wanted to calculate the probability of a student in the class having green as his or her favorite color, we could not use theoretical probability. We would have to collect data on the favorite colors of each member of the class and use experimental probability.

Determine whether theoretical or experimental probability would be appropriate for each of the following. Explain your reasoning:

1. What is the probability of someone tripping on the stairs today between first and second periods?
2. What is the probability of rolling a 3 on a six-sided die, then tossing a coin and getting a head?
3. What is the probability that a student will get 4 of 5 true false questions correct on a quiz?
4. What is the probability that a student is wearing exactly four buttons on his or her clothing today?

### Probability Homework: Experimental vs. Theoretical


- 1) A baseball collector checked 350 cards in case on the shelf and found that 85 of them were damaged. Find the experimental probability of the cards being damaged. Show your work.
- 2) Jimmy rolls a number cube 30 times. He records that the number 6 was rolled 9 times. According to Jimmy's records, what is the experimental probability of rolling a 6? Show your work.
- 3) John, Phil, and Mike are going to a bowling match. Suppose the boys randomly sit in the 3 seats next to each other and one of the seats is next to an aisle. What is the probability that John will sit in the seat next to the aisle?

- 4) In Mrs. Johnson's class there are 12 boys and 16 girls. If Mrs. Johnson draws a name at random, what is the probability that the name will be that of a boy?
- 5) Antonia has 9 pairs of white socks and 7 pairs of black socks. Without looking, she pulls a black sock from the drawer. What is the probability that the next sock she pulls out will also be black?
- 6) Lenny tosses a nickel 50 times. It lands heads up 32 times and tails 18 times. What is the experimental probability that the nickel lands tails?
- 7) A car manufacturer randomly selected 5,000 cars from their production line and found that 85 had some defects. If 100,000 cars are produced by this manufacturer, how many cars can be expected to have defects?

The following advertisement appeared in the Sunday paper:

**Chew DentaGum!**

4 out of 5 dentists surveyed agree that chewing DentaGum after eating reduces the risk of tooth decay! So enjoy a piece of delicious DentaGum and get fewer cavities!



10 dentists were surveyed.

- 8) According to the ad, what is the probability that a dentist chosen at random does not agree that chewing DentaGum after meals reduces the risk of tooth decay?
- 9) Is this probability theoretical or experimental? How do you know?
- 10) Do you think that the this advertisement is trying to influence the consumer to buy DentaGum? Why or why not?

11) What could be done to make this advertisement more believable?

### Designing Simulations

**Simulation** is a way to model random events, such that simulated outcomes closely match real-world outcomes. By observing simulated outcomes, researchers gain insight on the real world. When designing a simulation, you need to make sure you understand and answer the following questions:

#### What is the problem being simulated?

- What are the possible outcomes?
- What is the probability of each outcome?
- Are there any assumptions?
- What question is the simulation trying to answer?

#### What random device will be used to do the simulation, and how will it be used?

- Will it be a coin, spinner, playing cards, number cube, random digit table, or the random number generator?
  - Coin: What does each side of the coin represent?
  - Spinner: What does each section represent?
  - Playing Cards: What does each color, number, or suit represent?
  - Dice: What does each face represent?
  - Random Digit Table: What does each number represent?
  - Random Number Generator: What does each number represent?

#### What is one trial in this simulation?

- Coin: How many times do you need to flip?
- Spinner: How many times do you need to spin?
- Playing Cards: How many cards do you need to choose?
- Dice: How many times do you need to throw?
- Random Digit Table: How many digits do you need to look at?
- Random Number Generator: How many digits do you need to look at?

#### How many trials will be conducted?

- Will the process be repeated 10, 20, 30 times?

#### What are the results of the simulated trials?

- State the specific results of YOUR trial – you should get a fraction.

*My results show that \_\_\_\_\_ out of \_\_\_\_\_ were \_\_\_\_\_.*

#### What predictions can be made based on these results?

- This should include your conclusion based on the simulation you conducted.

*Based on my results the chances of \_\_\_\_\_ are \_\_\_\_\_.*

**Generating Random Numbers**

Random numbers can be quickly and easily generated by using the graphing calculator.

Let's start our investigation by looking at generating random integers.

(The TI-84+ is being used on this page.)

**Generating Random Integers:**

```
MATH NUM CPX [PRB]
1:rand
2:nPr
3:nCr
4:!
5:randInt(
6:randNorm(
7:randBin(
```

Go to **MATH** → **PRB**  
Choose #5 **randInt**(

```
randInt(1,25)
13
2
25
```

From the home screen, enter the smallest value needed, followed by the largest value. Hitting **ENTER** will generate the random integers. (Random values may repeat.) This example generates random numbers from 1 to 25 (good for Bingo).

```
randInt(1,25,3)
(8 13 11)
(6 3 25)
(6 4 24)
```

Adding a third parameter indicates the number of random integers that will appear on the screen at one time.

**Generating Random Numbers (not integers):**

```
MATH NUM CPX [PRB]
1:rand
2:nPr
3:nCr
4:!
5:randInt(
6:randNorm(
7:randBin(
```

Go to **MATH** → **PRB**  
Choose #1 **rand**

```
rand
.2795027564
```

The **rand** command will create a random number **between 0 and 1**.

```
rand*15
1.902000065
```

To generate a random number between 0 and 15, enter **rand\*15**.

```
rand(10)*15→L1
(5.192184537 6...
```

This last entry shows how to generate a list of 10 random numbers between 0 and 15 and store them in List 1.

**Generating Random Integers in Lists:**

```
MATH NUM CPX [2/3]
1:rand
2:nPr
3:nCr
4:!
5:randInt(
6:randNorm(
7:randBin(
```

Go to **MATH** → **PRB**  
Choose #5 **randInt(**

```
randInt(0,1,100)
→L1
(1 0 1 1 0 1 1 ...
```

Enter **randInt** followed by the smallest value in the desired range, the largest value, and the number of terms needed. The results are stored (**STO**) into List 1.  
This example stores 100 random integers from 0 to 1 in L1.

L1		L3	2
1	-----	-----	
0			
1			
1			
0			
1			
1			

```
L2=randInt(0,1,...
```

**OR**, from the list screen, arrow up onto **L2**, and type **randInt(0,1,100)**. Hit **ENTER**.  
Be sure to enter the third parameter so the calculator will know "how many" numbers to place in the list.

L1	L2	L3	2
1	0	-----	
0	0		
1	0		
1	0		
0	0		
1	1		
1	0		

```
L2(1)=0
```

Such lists can be used to simulate the toss of one (or more) fair coin(s). The number of entries represents the number of tosses. An even random number represents heads, while an odd number represents tails.  
If tossing one coin, use **sum** command to count the number of heads, where heads are 1, and tails are 0.

**2nd STAT - MATH - #5 sum**

NAMES OPS	MATH	sum(L1)
1:min(		
2:max(		
3:mean(		
4:median(		
5:sum(		50
6:Prod(		
7:stdDev(		

**Re-Seeding the Random Number Generator:** Calculators (and computers) are not capable of creating "truly random" numbers. They create what are called "pseudo-random" numbers, meaning they use a formula to create the values. To engage this formula, the calculator uses a starting value, called a "seed", and then creates the random numbers based upon this seed. If two calculators start with the same seed value, they will generate the same sequence of random values.

```
5→rand
5
```

If you wish, you can control the starting "seed" value. To seed the random generator, choose a seed value and store it into the **rand** command.

```
5→rand
5
rand
.7278038625
```

Now, start generating your random values. If you feed two calculators the same seed value, they will each produce the same result when **rand** is entered.

```
rand
Done
.9435974025
```

After running a **RESET (DEFAULTS)** the calculator will return to using its default seed value. Engaging **rand** will always produce the value seen above.

If you wish to ensure that each student in the class has a different set of random numbers, assign a different number to each student as their seed value. You could also have the students enter their birth date as the random seed (04021990), assuming no two students have the same birth date.

## Random digit table

### Line

101	19223	95034	05756	28713	96409	12531	42544	82853
102	73676	47150	99400	01927	27754	42648	82425	36290
103	45467	71709	77558	00095	32863	29485	82226	90056
104	52711	38889	93074	60227	40011	85848	48767	52573
105	95592	94007	69971	91481	60779	53791	17297	59335
106	68417	35013	15529	72765	85089	57067	50211	47487
107	82739	57890	20807	47511	81676	55300	94383	14893
108	60940	72024	17868	24943	61790	90656	87964	18883
109	36009	19365	15412	39638	85453	46816	83485	41979
110	38448	48789	18338	24697	39364	42006	76688	08708
111	81486	69487	60513	09297	00412	71238	27649	39950
112	59636	88804	04634	71197	19352	73089	84898	45785
113	62568	70206	40325	03699	71080	22553	11486	11776
114	45149	32992	75730	66280	03819	56202	02938	70915
115	61041	77684	94322	24709	73698	14526	31893	32592
116	14459	26056	31424	80371	65103	62253	50490	61181
117	38167	98532	62183	70632	23417	26185	41448	75532
118	73190	32533	04470	29669	84407	90785	65956	86382
119	95857	07118	87664	92099	58806	66979	98624	84826
120	35476	55972	39421	65850	04266	35435	43742	11937
121	71487	09984	29077	14863	61683	47052	62224	51025
122	13873	81598	95052	90908	73592	75186	87136	95761
123	54580	81507	27102	56027	55892	33063	41842	81868
124	71035	09001	43367	49497	72719	96758	27611	91596
125	96746	12149	37823	71868	18442	35119	62103	39244

## Simulations with a Random Digit Table

### Examples:

1. Suppose you wish to roll two dice a total of 5 times and keep track of the totals. You don't have any dice, but you do have access to a random digit table. Explain how you could simulate the rolls of two dice using the random digit table. Perform the simulation and record your results.

The easiest way to roll 2 dice and record the sums is to use 2 single digit numbers 1 – 6, and ignore 7 – 9. If you represent the sums 02 – 12, the probabilities will not be represented, since there is a higher probability of getting a sum of 7 than 12.

Starting on line 104 of the random digit table:

**52711 38889 93074 60227 40011 85848 48767 52573**

Rolling two dice 5 times gives the following results:

1<sup>st</sup> roll: 5 & 2, total = 7

2<sup>nd</sup> roll: 1 & 1, total = 2

3<sup>rd</sup> roll: 3 & 3, total = 6

4<sup>th</sup> roll: 4 & 6, total = 10

5<sup>th</sup> roll: 2 & 2, total = 4

2. At the start of this season, Major League Baseball fans were asked which American League Central team would be most likely to win the division this year. The table below gives the results of the poll.

Most Likely to Win AL Central					
Team	Chicago	Cleveland	Detroit	Kansas City	Minnesota
Probability	0.14	0.23	0.33	0.02	0.28

Using the random digit table, starting on line 117, simulate the results when asking 10 fans who they think will win the AL Central.

Use the probabilities to assign digits 00-99 as follows: Chicago: 00 – 13, Cleveland: 14 – 36; Detroit: 37 – 69; KC: 70 – 71; Minnesota: 72 – 99.

Starting on line 104 of random digit table:

**38167 98532 62183 70632 23417 26185 41448 75532**

The first ten people selected are 38, 16, 79, 85, 32, 62, 18, 37, 06, 32. Results of these ten give the following probabilities: Chicago: 10%; Cleveland: 40%; Detroit: 30%; KC: 0%; Minnesota: 20%

Note: assigning number 001 – 100, would require selecting 3 digits for each person and discarding numbers > 100.

3. A teacher must choose two students from a group of ten to participate in a certain activity. To avoid favoritism, she assigns numeric labels to each of her students as follows:

Number	Student	Number	Student
0	Amanda	5	Lynn
1	Bill	6	Malcombe
2	Daniel	7	Neal
3	Emilio	8	Samantha
4	Jacob	9	Tracy

Beginning on line 102 of the Table of Random Digits, the first two digits are 73. The two students chosen for the activity would be Neal (#7) and Emilio (#3).

Suppose the teacher wanted to select *four* students from the group of students in the table. The first 4 numbers are 7, 3, 6, 7, but 7 has already been chosen so we need to move to the next number, which is 6. That has also been chosen, so we pick yet another one: 4. The students are Neal (#7), Emilio (#3), Malcombe (#6), and Jacob (#4).

4. Now consider a group of 100 students. The best way to use 100 numbers is to assign a 2 digit numbers 00 – 99 to each student. (If you use three digit numbers 001-100 you have to skip a lot of numbers in the random digit table to pick 5.) Using the table of random digits starting at line 106, the five labels would you select are 68, 41, 73, 50, 13.

5. To gather data on a 1200-acre pine forest in Louisiana, the U.S. Forest Service laid a grid of 1410 equally spaced circular plots over a map of the forest. A ground survey visited a sample of 10% of these plots. Starting on line 104 of random digit table, choose the first 5 plots.

You'll need 4 digit numbers 0001 – 1410 (since you have extra numbers, there's no need to use 0000, but you could use 0000-1409 if desired). The first five numbers chosen are bolded. Note that numbers that are not between 0000 and 1410 are skipped.

5271	1388	8993	0746	0227	4001	1858	4848	7675	2573
9559	2940	0769	9719	1481	6077	9537	9117	2975	9335
6841	7350	1315							

Now you try:

6. A medical study of heart surgery investigates the effect of a drug called a beta-blocker on the pulse rate of the patient during surgery. The pulse rate will be measured at a specific point during the operation. The investigators will use 20 patients facing heart surgery as subjects. You have a list of these patients, numbered 1 to 20, in alphabetical order. Half of these subjects will receive the beta-blocker during surgery. Use line 107 from the random digits table to determine which patients will receive the treatment.

7. There are 120 students in the BETA club. The national convention is to be held in Las Vegas this year, but only 20 students are interested in attending. Because of expenses, Mr. Ed Visor can only take 3 students. She has asked you to help him randomly pick the 3 students who will get to go to Las Vegas. The twenty students who wish to go are:

Moore	Phillips	Barnes	Shaw	Cook	
Garris	Jones	Allen	Scott	Flynn	
Norris	Jacobs	Long	Edwards	Bennett	
Dixon	Alex	ThomasYoung	Glenn		

Determine a method to select the 3 students who will get to go to Las Vegas, then use line 101 to pick the students.



## Practice – Simulations with a Random Digit Table

1. Every person is born on a different day of the month. Some people are born on the 1st and some people are born as late as the 31st. How many people must you go through until you find two that were born on the same day of the month? Simulate this one time using the random digit table starting at line 101. (Ignore the fact that people are not equally likely to be born on all days. It is more likely you were born on the 17th than the 31st since all months have a 17th but not all months have a 31st.)
2. Suppose that 80% of a school's student population is in favor of eliminating final exams.
  - a) Explain how could you assign digits from a random digit table to simulate this situation?
  - b) Suppose you ask 10 students if they would like to eliminate final exams. Simulate a random selection of 20 students and record how many of the 20 are in support of eliminating final exams. Use the random digit table starting at line 110.
3. Suppose that students at a particular college are asked about their class rank when they were in high school. The table below shows what they said.

Class Rank	Top 10%	Top 10% to 25%	Top 25% to 50%	Bottom 50%
Prob.	0.2	0.4	0.3	

- a) What must the probability be for the bottom 50%?
  - b) Explain how you could assign digits to carry out a simulation for this situation.
  - c) Using your set up, perform a simulation using the random digit table starting at line 118. Use 20 students in your simulation and record your results.
4. Suppose the grades for students in your math course were distributed as shown in the table.

Grade	A	B	C	D or F
Prob.	0.20	0.29	0.35	0.16

- a) Explain how you could assign digits to simulate the grades of randomly chosen students.
  - b) Simulate the grades for 30 students using the random digit table starting at line 113. Record your results.
  - c) How closely did your simulation match the actual distribution?
5. How many five card poker hands must you be dealt in order to get a hand with two cards that have matching values? (For example, the 7 of hearts and 7 of diamonds have matching values.)
    - a) Explain how you will assign digits for this situation.
    - b) Perform the simulation one time using the random digit table starting at line 105, and state how many five-card hands it took for you to get your first hand with two cards that match.
  6. Suppose we have a class of 30 students and you are wondering what the chances are that there is at least one pair of students who have the same birthday. Assume that there are 365 days in a year.
    - a) Explain how could you assign digits from a random digit table to simulate this situation?
    - b) Perform this simulation one time using the random digit table starting at line 111, and record whether or not there was a match in the class of 30 students.

## Design a Simulation

Read the following situations and determine if/how each of random devices could be used to conduct a simulation.

1. A high school basketball player makes 50% of his shots from the three-point line. If he takes 13 shots during a game predict the number of baskets he will make.

Random Device	Can it be used?	How?
Coin		
Spinner		
Playing Cards		
Dice		
Random Number Table		
Random Number Generator		

2. In a family of 5, what is the probability that three of the members are male?

Random Device	Can it be used?	How?
Coin		
Spinner		
Playing Cards		
Dice		
Random Number Table		
Random Number Generator		

3. In a random survey, 68% of high school students report that they have less than an hour of homework on any given night. What is the probability that a high school student selected at random will have less than an hour of homework on any given night.

Random Device	Can it be used?	How?
Coin		
Spinner		
Playing Cards		
Dice		
Random Number Table		
Random Number Generator		

4. The probability that a student gets an A or B on a Chapter Test is  $\frac{1}{3}$ . What is the probability that a student chosen at random does not get an A or a B on the test?

Random Device	Can it be used?	How?
Coin		
Spinner		
Playing Cards		
Dice		
Random Number Table		
Random Number Generator		

Questions for review:

1. Use the table on the left to answer the following questions:

a) If each of the sections on the spinner are the same size, what is the theoretical probability that any given number will be spun?

b) What was the experimental probability of how many times a 4 was actually rolled using the table?

c) Theoretically if you spin this spinner 50 times, how many times would you expect to roll the number 3?

d) How many times did you actually roll the number 3 in the experiment?

Number on Spinner	Frequency
1	4
2	5
3	2
4	6
5	2

2. Your sock drawer is a mess! You just shove all of your socks in the drawer without worrying about finding matches. Your aunt asks how many pairs of each color you have. You know that you have 32 pairs of socks, or 64 individual socks in four different colors: white, blue, black, and tan. You do not want to count all of your socks, so you randomly pick 20 individual socks and predict the number from your results.

Sock color	white	blue	black	tan
Number	12	1	3	4

a) Find the experimental probability of each

P(white) = \_\_\_\_\_ P(blue) = \_\_\_\_\_ P(black) = \_\_\_\_\_ P(tan) = \_\_\_\_\_

b) Based on your experiment, how many socks of each color are in your drawer?

(white) = \_\_\_\_\_ (blue) = \_\_\_\_\_ (black) = \_\_\_\_\_ (tan) = \_\_\_\_\_

c) Based on your results, how many pairs of each sock are in your drawer?

(white) = \_\_\_\_\_ (blue) = \_\_\_\_\_ (black) = \_\_\_\_\_ (tan) = \_\_\_\_\_

d) Your drawer actually contains 16 pairs of white socks, 2 pairs of blue socks, 6 pairs of black socks, and 8 pairs of tan socks. How accurate was your prediction?

3. Teachers A, B, and C all teach Math 2. Because their classrooms are different sizes, students have a 25% chance of getting teacher A, a 35% chance of getting teacher B, and a 40% chance of getting teacher C. Eight girls are close friends and wonder whether they could all be in the same class. Explain how you could set up a simulation to determine the probability of this occurring.

4. Lesley estimates that she has a 75% chance of passing physics and an 80% chance of passing English. Assuming that "passing English" and "passing Physics" are independent events, what is the probability that Lesley-Anne will pass only one of these two subjects?

5. If a satellite launch has a 97% chance of success, what is the probability of three consecutive successful launches?

6. In a survey at a football game, 50 of 75 male fans and 40 of 50 female fans said that they favor the new team mascot. If 1 male and 1 female are randomly selected, what is the probability that both favor the new mascot?
7. Find the probability of selecting the following:
- |                                      | <u>With replacement</u> | <u>Without replacement</u> |
|--------------------------------------|-------------------------|----------------------------|
| a) A club, then a spade              |                         |                            |
| b) A queen, then an ace              |                         |                            |
| c) A face card, then a 6             |                         |                            |
| d) A 10, then 2                      |                         |                            |
| e) A king, then a queen, then a jack |                         |                            |
| f) Three hearts in a row             |                         |                            |
8. One bag contains 2 green marbles and 4 white marbles, and a second bag contains 3 green marbles and 1 white marble. If Trent randomly draws one marble from each bag, what is the probability that they are both green?
9. A math teacher is randomly distributing 15 rulers with centimeter labels and 10 rulers without centimeter labels. What is the probability that the first ruler she hands out will have centimeter labels and the second ruler will NOT have labels?
10. Andrea is a very good student. The probability that she studies and passes her mathematics test is 85%. If the probability that Andrea studies is 94%, find the probability that Andrea passes her mathematics test, given that she has studied.
11. The probability that Janice smokes is 30%. The probability that she smokes and develops lung cancer is 28%. Find the probability that Janice develops lung cancer, given that she smokes.
12. The probability that Sue will go to Mexico in the winter and to France in the summer is 0.40. The probability that she will go to Mexico in the winter is 0.60. Find the probability that she will go to France this summer, given that she just returned from her winter vacation in Mexico.
13. A penny and a nickel are tossed. Find the probability that the penny shows heads, given that the nickel shows heads.
14. A die is tossed. Find  $P(\text{less than 5} \mid \text{even})$ .
15. A box contains three blue marbles, five red marbles, and four white marbles. If one marble is drawn at random, find:
- $P(\text{blue} \mid \text{not white})$
  - $P(\text{not red} \mid \text{not white})$

16. Sean pulls two coins out of his pocket randomly without replacement. If his pocket contains one nickel, one dime, and one quarter, what is the probability that he pulled more than 20 cents out of his pocket? Justify your work by creating a tree diagram or sample space.

17. Amy, Breonna, Allison, and Bella are trying to decide who will make dinner and who will wash the dishes afterward. They randomly pull two names out of a hat to decide – the first name drawn will make dinner and the second will do the dishes. Find the probability that the two people pulled will have first names beginning with the same letter. Assume the same person cannot be picked for both. Justify your work by creating a tree diagram or sample space.

18. A four-sided die, in the shape of a tetrahedron, is rolled twice and the number rolled is recorded each time.

a) Draw a tree diagram that shows the sample space,  $S$ , of this experiment. How many elements are in  $S$ ?

b) Let  $E$  be the event of rolling two numbers that have an odd product. List all of the elements of  $E$  as ordered pairs.

c) What is the probability that the two rolled numbers have a product that is odd?

d) What is the probability that the two rolled numbers have a product that is even?

19. A poll finds that 72% of Jacksonville consider themselves football fans. If you randomly pick two people from the population, what is the probability the first person is a football fan and the second is as well?

20. The probability that a student owns a car is 0.65, and the probability that a student owns a computer is 0.82. If the probability that a student owns both is 0.55

a) What is the probability that a randomly selected student owns a car or computer?

b) What is the probability that a randomly selected student does not own a car or computer?

21. In a statistics class there are 18 juniors and 10 seniors; 6 of the seniors are females and 12 of the juniors are males. If a student is selected at random, find the probability of selecting the following:

a)  $P(\text{a junior or a female})$

b)  $P(\text{a senior and a female})$

c)  $P(\text{a junior} \mid \text{senior})$

22. At a particular school with 200 male students, 58 play football, 40 play basketball and 8 play both.

a) Find the probability that a randomly selected male student plays basketball or football.

b) Find the probability that a randomly selected male student plays neither sport.

23. A flashlight has 6 batteries, 2 of which are defective. If 2 are selected at random without replacement, find the probability that both are defective.

24. At a large university, the probability that a student takes calculus and is on the dean's list is 0.042. The probability that a student is on the dean's list is 0.21. Find the probability that a student takes calculus, given that he or she is on the dean's list.

25. If  $P(A) = 0.26$  and  $P(B) = 0.41$  and  $P(A \cap B) = 0.1$ , are the events independent?

26. If  $P(G) = 0.42$ ,  $P(M) = 0.33$  and  $G$  and  $M$  are independent, what's the probability of  $G$  and  $M$ ?

27. Use the chart to answer the questions. You select one person who has taken the written drivers test.

	Pass	Fail	Total
Men	50	30	80
Women	40	20	60
Total	90	50	140

a) Find the probability that you select a woman

b) Find the probability that you select a woman, given that she passed.

c) Find the probability that you select a failure, given it was a man

d) Are being male and passing the written test independent?

28. What is the probability of getting doubles knowing that the sum is greater than four?