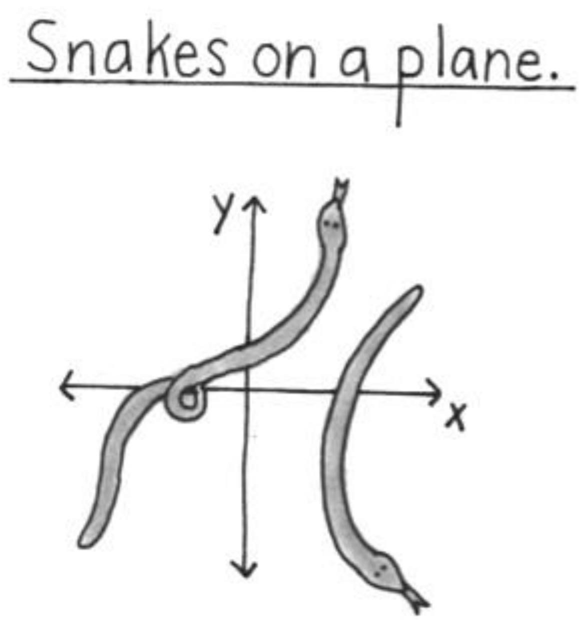
Name:______________________________

Period: _____

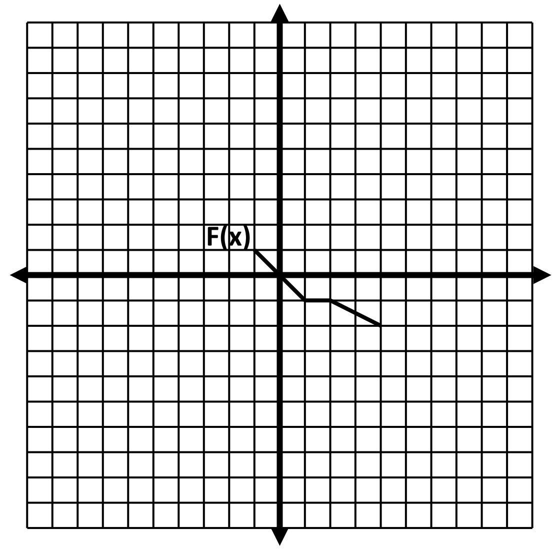
Unit 1B
FRED FUNCTION



Transformations with Fred Functions

Translations

To the right is the graph of **"Fred"**.



1. How do we know that Fred is a function?

2. According to the graph, what is Fred's domain?

3. According to the graph, what is Fred's range?

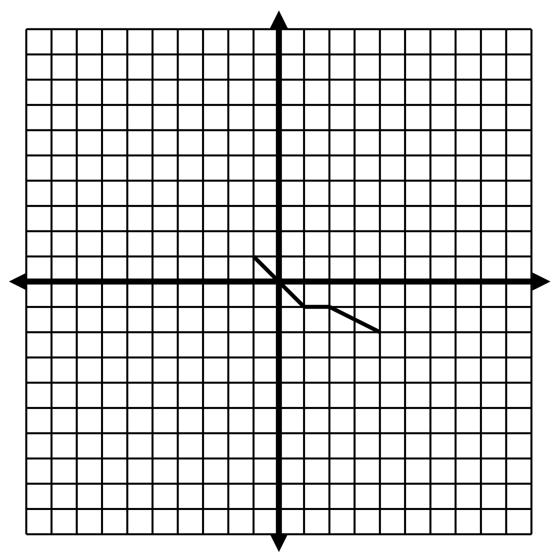
4. Let's explore the points on Fred.
   a. The key points (or "characteristic" points) are points like corner points and endpoints that help us to graph a function. Write the coordinates of Fred's key points here:

   b. Use the graph of Fred to evaluate the following.

   F(1) = ______ F(–1) = ______ F(______) = –2 F(5) = _______

I. Remember that F(x) is another name for the y-values, so we can write **y = F(x)**. Use Fred's graph to fill in the table with the y-values that correspond to the given x-values.

x	F(x)	y =F(x) +4
–1		
1		
2		
4		



Now let's graph Freddie Jr.: **y = F(x) + 4**. Complete the table below for this new function and then graph each of the (x, y) coordinates for Freddie Jr. on the coordinate plane above.

1. What type of transformation maps Fred, F(x), to Freddie Jr., F(x) + 4? (Be specific.)

2. How did this transformation affect the x-values? *(Hint: Compare the key points of Fred and Freddie Jr.)*

3. How did this transformation affect the y-values? *(Hint: Compare the key points of Fred and Freddie Jr.)*

4. In y = F(x) + 4, how did the "+4" affect Fred's domain?

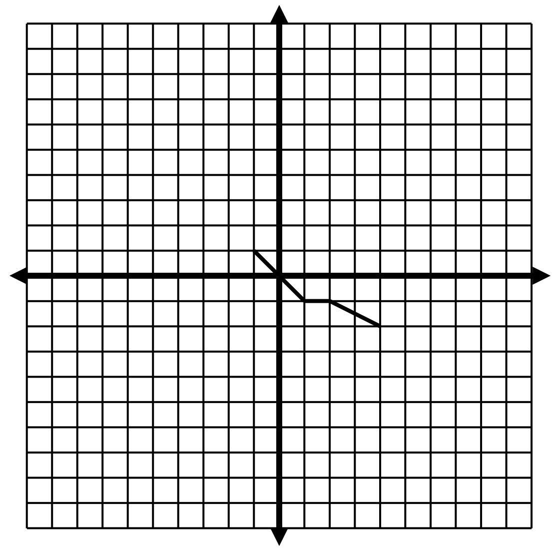
   How did the "+4" affect Fred's range?

Transformations with Fred Functions

II. Suppose Freddie Jr's equation is: **y = F(x) – 3**. Complete the table below for this new function and then graph Freddie Jr. on the coordinate plane above.

y = F(x) – 3

x	F(x)	y = F(x) - 3
–1		
1		
2		
4		



1. What type of transformation maps Fred, F(x), to Freddie Jr., F(x) - 3? (Be specific.)

2. How did this transformation affect the x-values? *(Hint: Compare the key points of Fred and Freddie Jr.)*

3. How did this transformation affect the y-values? *(Hint: Compare the key points of Fred and Freddie Jr.)*

4. In y = F(x) - 3, how did the "-3" affect Fred's domain?

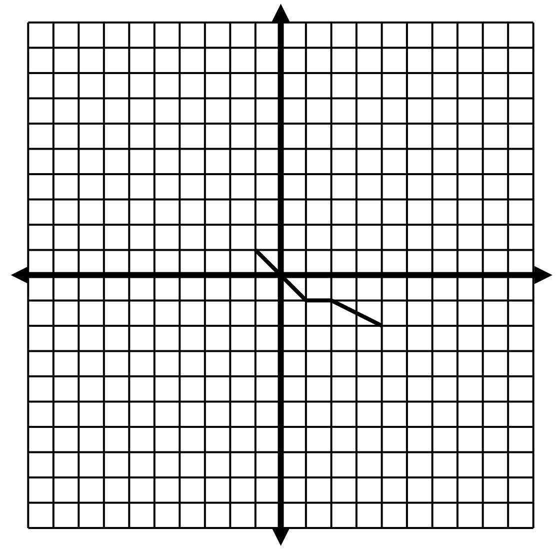
   How did the "-3" affect Fred's range?

III. Checkpoint: Describe the effect on Fred for the following functions.

Equation	**Effect on Fred's graph**
Example: y=F(x) + 18	Translate up 18 units
1. y = F(x) – 100	
2. y = F(x) + 73	
3. y = F(x) + 32	
4. y = F(x) – 521	

Transformations with Fred Functions

IV. Suppose Freddie Jr's equation is: **y = F(x + 4)**.



1. Complete the table.

y = F(x + 4)

x	x + 4	y
–5	–1	1
	1	–1
	2	–1
	4	–2

2. On the coordinate plane above, graph the 4 ordered pairs (x, y). The first point is (–5, 1).

3. What type of transformation maps Fred, F(x), to Freddie Jr., F(x + 4)? (Be specific.)

4. How did this transformation affect the x-values? *(Hint: Compare the key points of Fred and Freddie Jr.)*

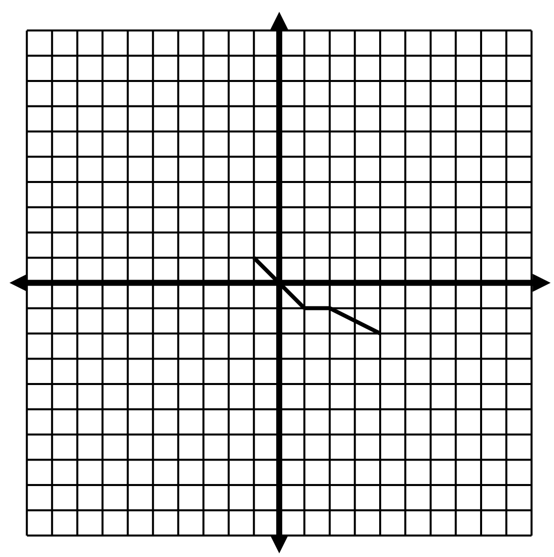
5. How did this transformation affect the y-values? *(Hint: Compare the key points of Fred and Freddie Jr.)*

6. In y = F(x + 4), how did the "+4" affect Fred's domain?

   How did the "+4" affect Fred's range?

V. Suppose Freddie Jr's equation is: **y = F(x – 3)**.

1. Complete the table.



y = F(x – 3)

x	x – 3	y
	–1	1
	1	-1
	2	-1
	4	-2

2. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st point should be (2, 1).*]

3. What type of transformation maps Fred, F(x), to Freddie Jr., F(x – 3)? (Be specific.)

4. How did this transformation affect the x-values? *(Hint: Compare the key points of Fred and Freddie Jr.)*

5. How did this transformation affect the y-values? *(Hint: Compare the key points of Fred and Freddie Jr.)*

6. In y = F(x – 3), how did the "- 3" affect Fred's domain?

   How did the "- 3" affect Fred's range?

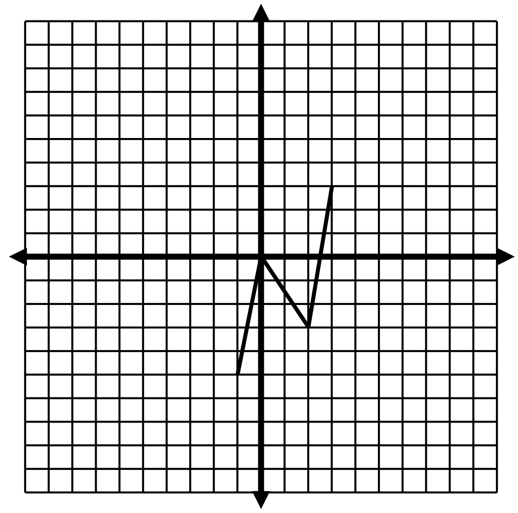
VI. Checkpoint: Describe the effect on Fred for the following functions.

Equation	**Effect on Fred's graph**
Example: y=F(x + 18)	Translate left 18 units
1. y = F(x – 10)	
2. y = F(x) + 7	
3. y = F(x + 48)	
4. y = F(x) – 22	
5. y = F(x + 30) + 18	

VII. Checkpoint: Write the equation that would have the following effect on Fred's graph.

Equation	**Effect to Fred's graph**
Example: y=F(x + 8)	Translate left 8 units
1.	Translate up 29 units
2.	Translate right 7
3.	Translate left 45
4.	Translate left 5 and up 14
5.	Translate down 2 and right 6

Transformations with Fred Functions

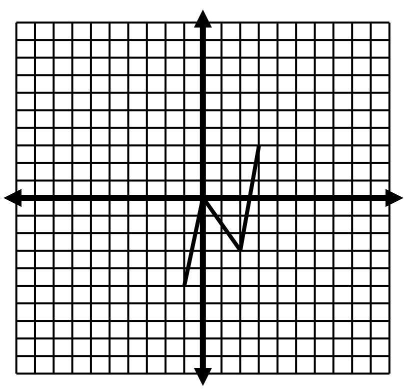
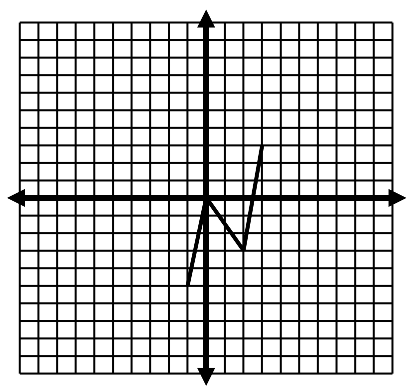
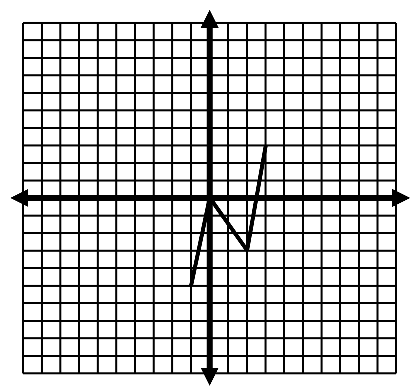


VIII. Now let's look at a new function.
Its notation is **H(x)**, and we will call it **Harry**.
Use Harry to demonstrate what you have learned so far about the transformations of functions.

1. What are Harry's characteristic points?

2. Describe the effect on Harry's graph for each of the following.
   a. H(x – 2) ____
   b. H(x) + 7 ____
   c. H(x+2) – 3 ____

3. Use your answers to questions 1 and 2 to help you sketch each graph *without using a table*.
   a. 44y = H(x – 2)
   b. y = H(x) + 7
   c. y = H(x+2) – 3

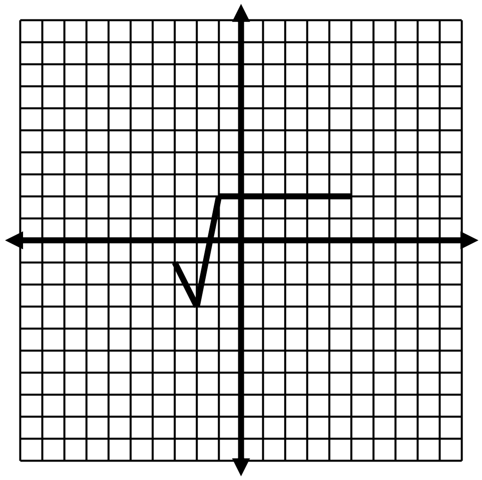




Transformations with Fred Functions

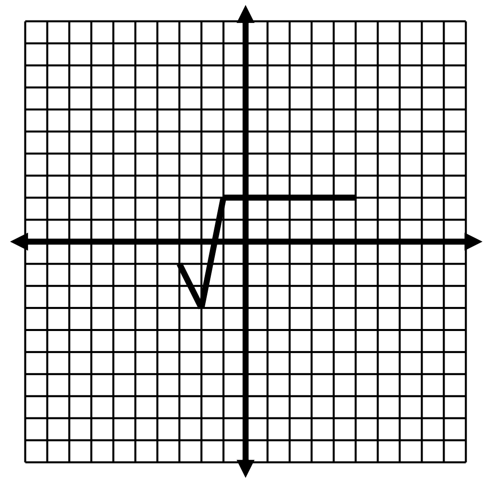
<u>HW Translations</u>

I. On each grid, **Ginger, G(x)** is graphed. Graph the given function.

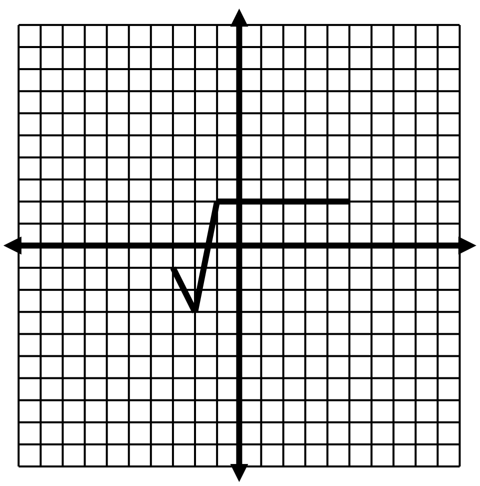
1. Graph: y = G(x) – 6



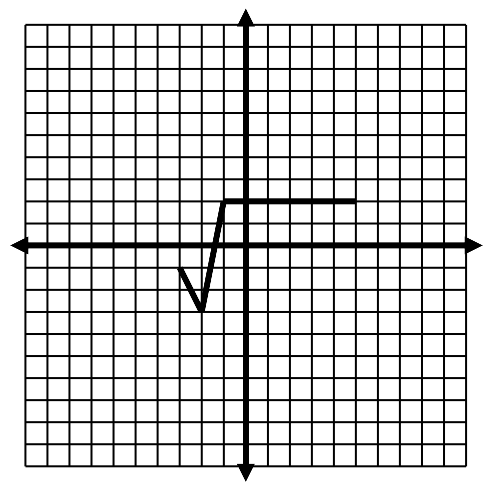
2. Graph: y = G(x + 6)



3. Graph: y = G(x + 2) + 5



4. Graph: y = G(x – 4) – 5



II. Using the understanding you have gained so far, describe the effect to Fred for the following functions.

Equation	Effect to Fred's graph
6. y = F(x) + 82	
7. y = F(x – 13)	
8. y = F(x + 9)	
9. y = F(x) – 55	
10. y = F(x – 25) + 11	

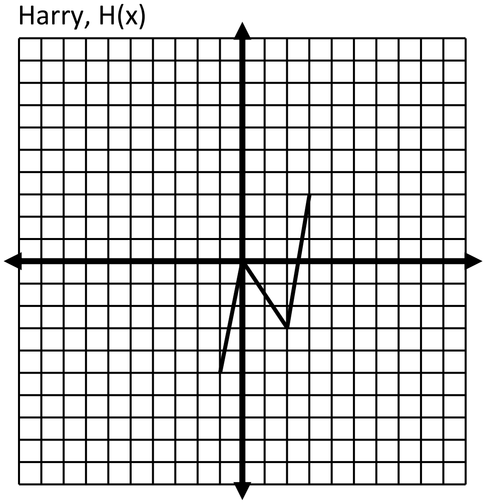
Transformations with Fred Functions

III. Using the understanding you have gained so far, write the equation that would have the following effect on Fred's graph.

Equation	Effect to Fred's graph
6.	Translate left 51 units
7.	Translate down 76
8.	Translate right 31
9.	Translate right 8 and down 54
10.	Translate down 12 and left 100

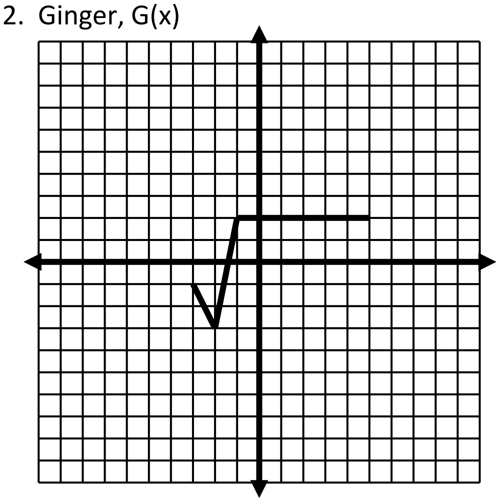
IV. Determine the domain and range of each parent function.

1. Harry, H(x)



Domain: ____________________
Range: ____________________

2. Ginger, G(x)



Domain: ____________________
Range: ____________________

V. Consider a new function, Polly, P(x).
Polly's Domain is $\{x| -2 \le x \le 2\}$. Its range is $\{y| -3 \le y \le 1\}$.

Use your understanding of transformations of functions to determine the domain and range of each of the following functions. (Hint: You may want to write the effect to Polly first.)

1. P(x) + 5

Domain: ____________________________
Range: ____________________________

2. P(x + 5)

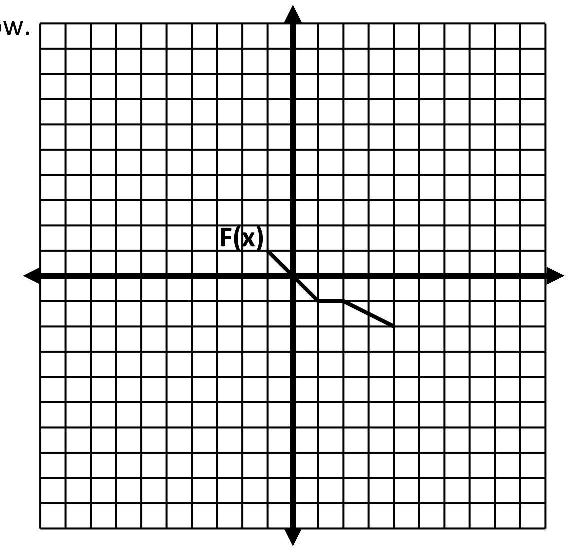
Domain: ____________________________
Range: ____________________________

Transformations with Fred Functions

Reflection and Dilation

Today we will revisit Fred, our "parent" function, and investigate transformations other than translations.

I. Remember that Fred is the function **y = F(x)**, graphed below.
Suppose that Freddie Jr. is **y = – F(x)**



1. Complete the table.

y = – F(x)

x	F(x)	y
–1	1	–1
1		
2		
4		

2. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st point should be (–1, –1).*]

3. What type of transformation maps Fred, F(x), to Freddie Jr., –F(x)? (Be specific.)

4. How did this transformation affect the x-values? *(Hint: Compare the key points of Fred and Freddie Jr.)*

5. How did this transformation affect the y-values? *(Hint: Compare the key points of Fred and Freddie Jr.)*

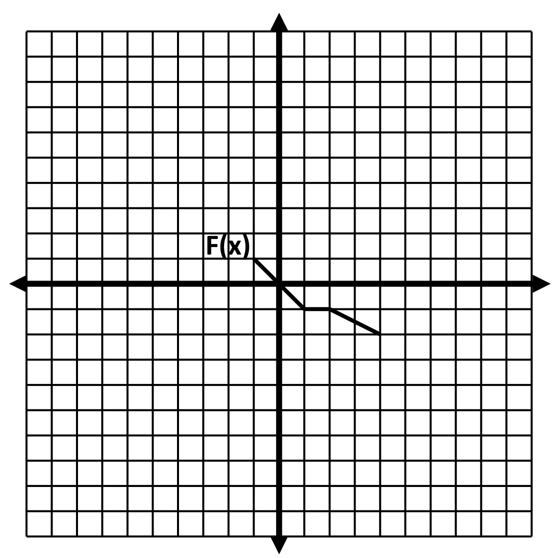
6. In y = – F(x), how did the negative sign in front of "F(x)" affect the graph of Fred? How does this relate to our study of transformations earlier this semester?

II. Now let's suppose that Freddie Jr. is **y = F(–x)**

1. Complete the table.

y = F(–x)

x	–x	y
	–1	
	1	
	2	
	4	



2. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st point should be (1, 1).*]

Transformations with Fred Functions

3. What type of transformation maps Fred, F(x), to Freddie Jr., F(–x)? (Be specific.)

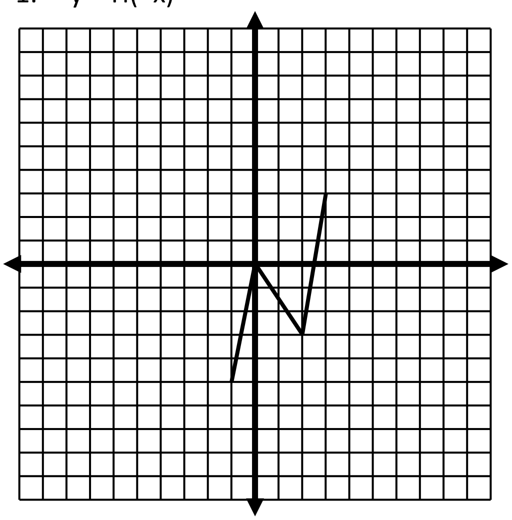
4. How did this transformation affect the x-values? *(Hint: Compare the key points of Fred and Freddie Jr.)*

5. How did this transformation affect the y-values? *(Hint: Compare the key points of Fred and Freddie Jr.)*

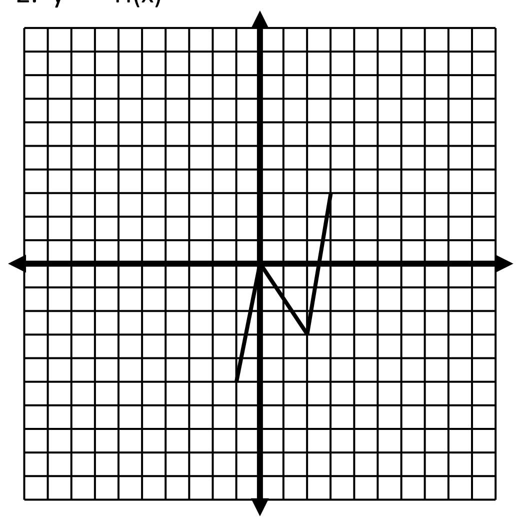
6. In y = F(–x), how did the negative sign in front of "x" affect the graph of Fred? How does this relate to our study of transformations earlier this semester?

III. Checkpoint: Harry is H(x) and is graphed on each grid below. Use Harry's characteristic points to graph Harry's children without making a table.

1. y = H(–x)



2. y = – H(x)

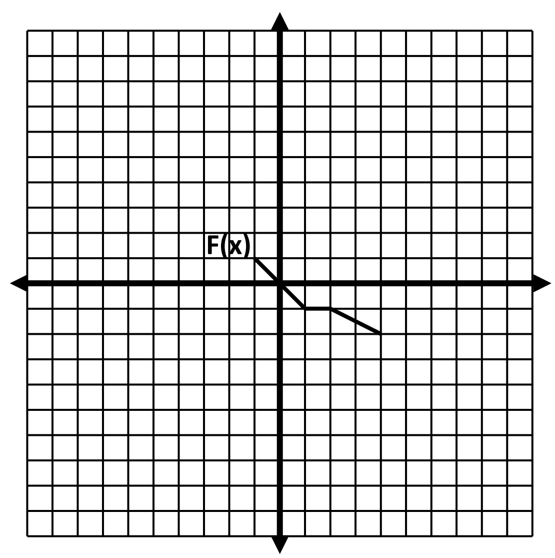


IV. Now let's return to Fred, **y = F(x)**, and suppose that Freddie Jr. is **y = 4 F(x)** .

1. Complete the table.

y = 4 F(x)

x	**F(x)**	**y**
–1		
1		
2		
4		



2. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st one should be (–1, 4).*]

3. How did this transformation affect the x-values? *(Hint: Compare the key points of Fred and Freddie Jr.)*

4. How did this transformation affect the y-values? *(Hint: Compare the key points of Fred and Freddie Jr.)*

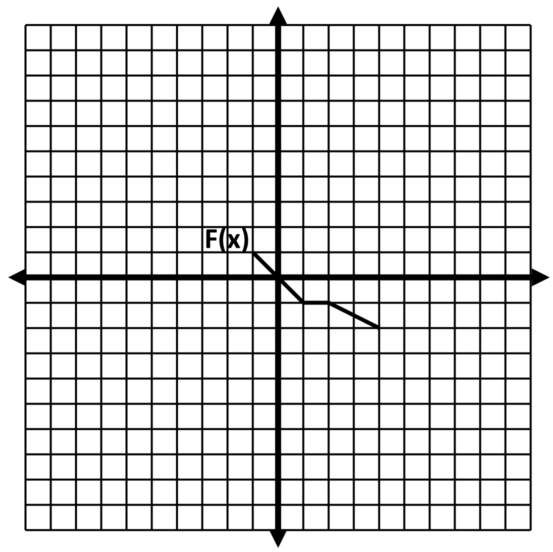
5. In y = 4 F(x), we multiplied "F(x)" times 4. How did that affect the graph of Fred? Is this one of the transformations we studied? If so, which one? If not, explain.

V. Now let's suppose that Freddie Jr. is **y = ½ F(x)**.

1. Complete the table.

y = ½ F(x)

x	F(x)	y
–1		
1		
2		
4		



2. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st one should be (–1, ½).*]

3. How did this transformation affect the x-values? *(Hint: Compare the key points of Fred and Freddie Jr.)*

4. How did this transformation affect the y-values? *(Hint: Compare the key points of Fred and Freddie Jr.)*

5. In y = ½ F(x), we multiplied "F(x)" times ½. How did that affect the graph of Fred? How is this different from the graph of y = 4 F(x) on the previous page?

VI. Checkpoint:

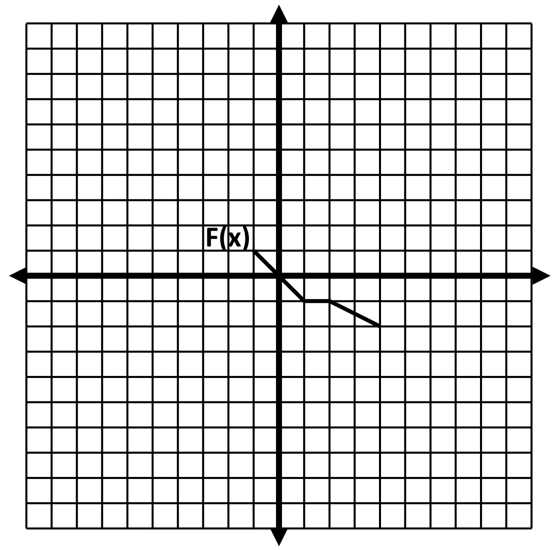
1. Complete each chart below. Each chart starts with the key points of Fred.

x	F(x)	3 F(x)
–1	1	
1	–1	
2	–1	
4	–2	

x	F(x)	¼ F(x)
–1	1	
1	–1	
2	–1	
4	–2	

2. Compare the 2nd and 3rd columns of each chart above. The 2nd column is the y-value for Fred. Explain how the graph of Fred changes when you multiply F(x) times a number.

VII. Now let's suppose that Freddie Jr. is **y = –3 F(x)**.



1. Complete the table.

y = –3 F(x)

x	F(x)	y
–1		
1		
2		
4		

2. On the coordinate plane above, graph the 4 ordered pairs (x, y).
   [*Hint: The 1st one should be (–1, –3).*]

3. Compare the 2nd and 3rd columns of the chart above. The 2nd column is the y-value for Fred. Explain how the graph of Fred changes when you multiply F(x) times a number AND the number is negative.

VIII. Checkpoint: Let's revisit Harry, H(x).

1. Describe the effect on Harry's graph for each of the following.

   Example: –5H(x) ____<u>Each point is reflected in the x-axis and is 5 times as far from the x-axis.</u>

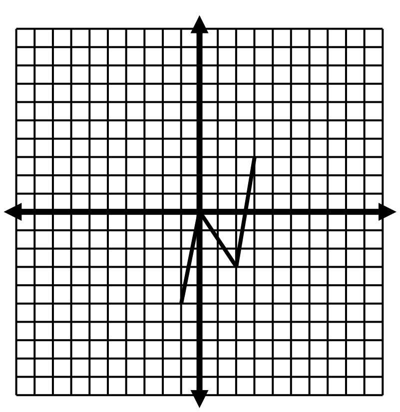
   d. 3H(x) ______________________________

   e. –2H(x) ______________________________

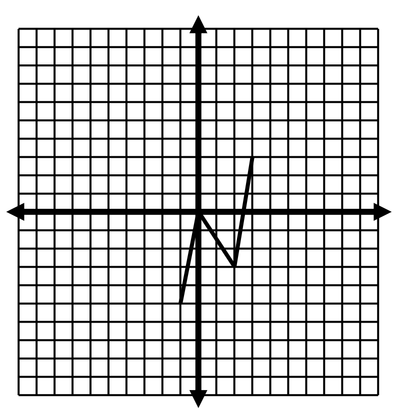
   f. $\frac{1}{2}$H(x) ______________________________

2. Use your answers to questions 1 and 2 to help you sketch each graph *<u>without using a table</u>*.

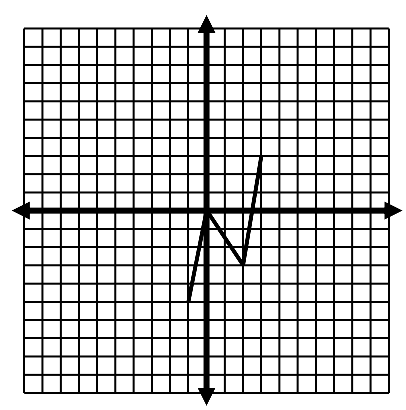
b. y = 2H(x)



b. y = –2H(x)

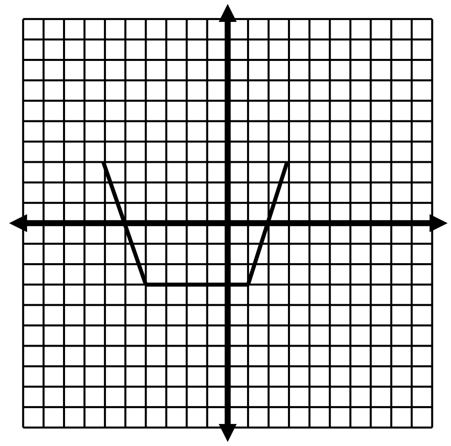


c. y = $\frac{1}{2}$H(x)



HW Reflection and Dilation

This is the function **Bowl, B(x)**.



1. List its characteristic points.

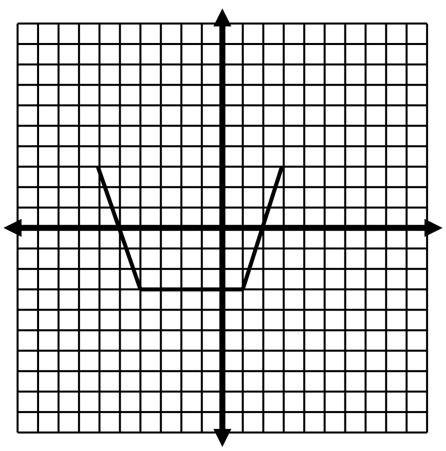
2. Are these the only points on the graph of Bowl? Explain.

3. What is the domain of Bowl?

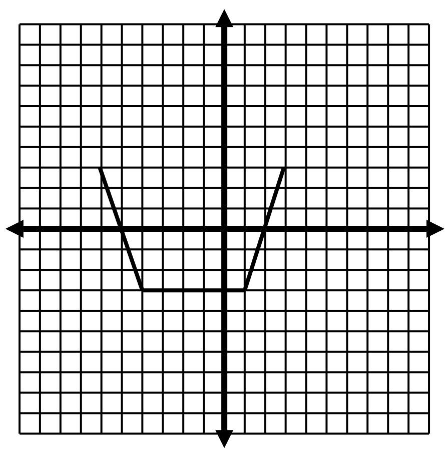
4. What is the range of Bowl?

For each of the following, list the effect on the graph of Bowl and then graph the new function.

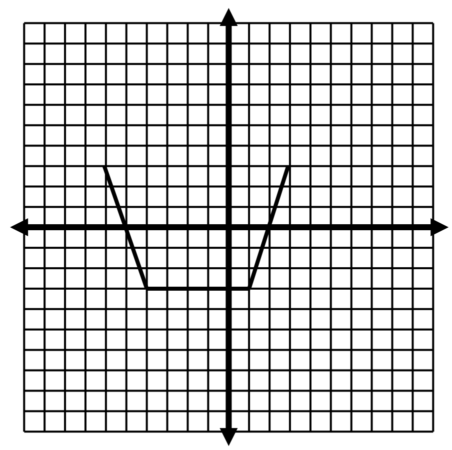
5. $y = B(-x)$



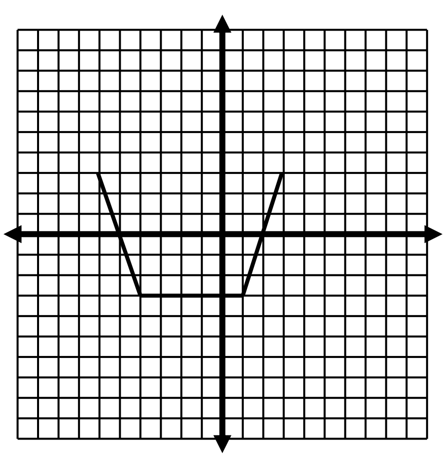
6. $y = -B(x)$



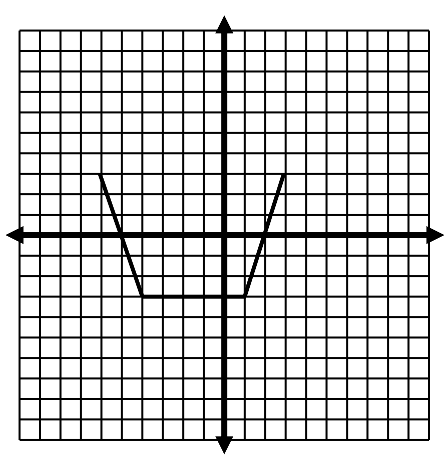
7. $y = \frac{1}{3}B(x)$



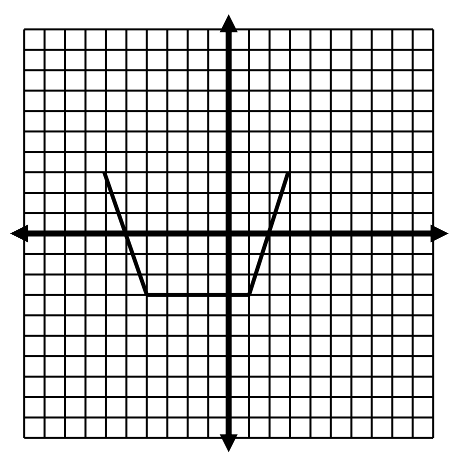
8. $y = 3B(x)$



9. $y = B(x - 3)$



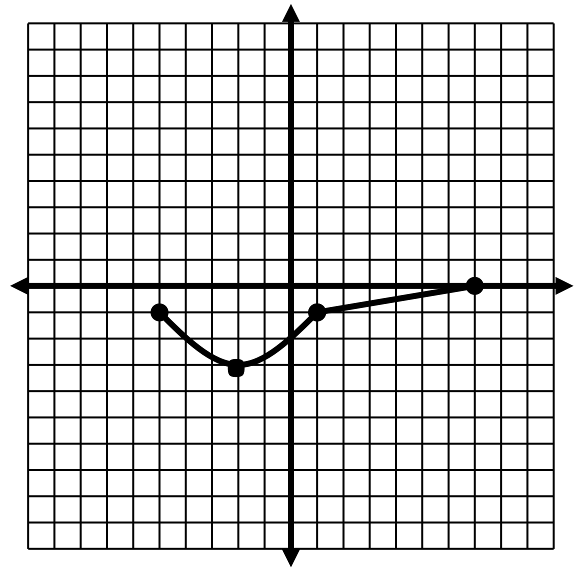
10. $y = B(x + 2) - 1$



Transformations with Fred Functions

Composite Transformations

The graph of **Dipper, D(x),** is shown.



List the characteristic points of Dipper.

____

What is different about Dipper from the functions we have seen so far?

Since Dipper is our original function, we will refer to him as the **parent function**. Using our knowledge of transformational functions, let's practice finding children of this parent.

Note: In transformational graphing where there are multiple steps, it is important to perform the translations last.

I. **Example:** Let's explore the steps to graph **Dipper Jr., 2D(x + 3) + 5**, without using tables.

Step 1. The transformations represented in this new function are listed below in the order they will be performed. (See note above.)

- Vertical stretch by 2 (each point moves twice as far from the x-axis).
- Translate left 3.
- Translate up 5.

Step 2. On the graph, put your pencil on the left-most characteristic point, (– 5, –1).

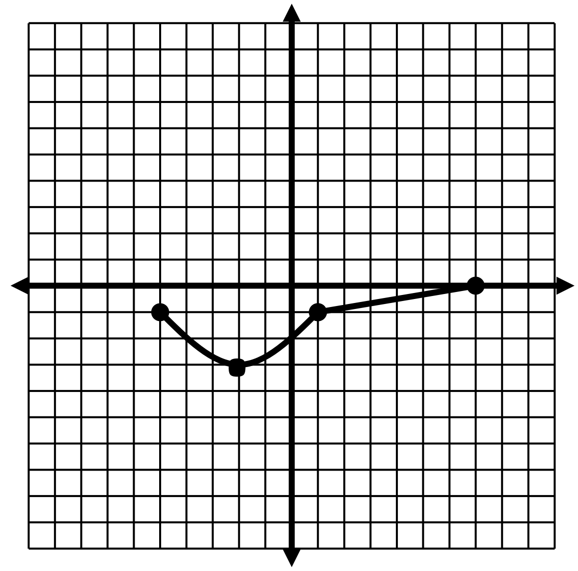
- Vertical stretch by 2 takes it to (– 5, –2). (Note that originally, the point was 1 unit away from the x-axis. Now, the new point is 2 units away from the x-axis.)
- Starting with your pencil at (– 5, –2), translate this point 3 units to the left. Your pencil should now be on (– 8, –2).
- Starting with your pencil at (– 8, –2), translate this point up 5 units. Your pencil should now be on (– 8, 3).
- Plot the point (– 8, 3). It is recommended that you do this using a different colored pencil.

Step 3. Follow the process used in Step 2 above to perform all the transformations on the other 3 characteristic points.

Step 4. After completing Step 3, you will have all four characteristic points for Dipper Jr. Use these to complete the graph of Dipper Jr. Be sure you use a curve in the appropriate place. Dipper is not made of segments only.

Transformations with Fred Functions

II. Dipper has another child named **Little Dip, – D(x) – 4**



Using the process in the previous example as a guide, graph Little Dip (without using tables).

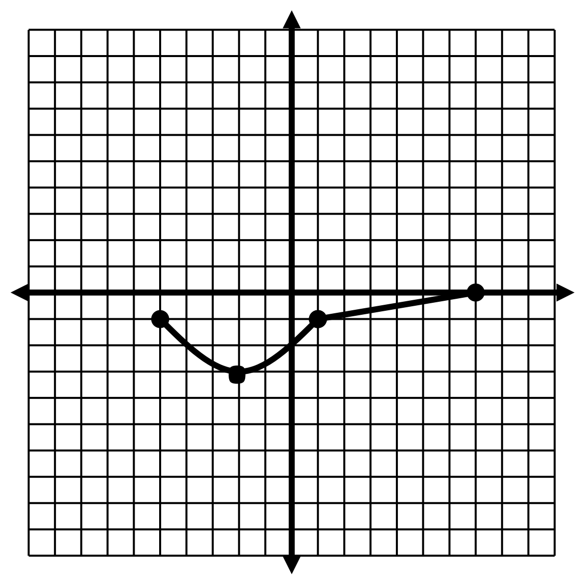
1. List the transformations needed to graph Little Dip. Remember to be careful with order.

- ____
- ____

2. Apply the transformations listed above to each of the four characteristic points.

3. Complete the graph of Little Dip using your new characteristic points from #2.

III. Dipper has another child named **Dipsy, 3 D(– x)**



Using the process in the previous example as a guide, graph Dipsy (without using tables).

1. List the transformations needed to graph Dipsy. Remember, to be careful with order.

- ____
- ____

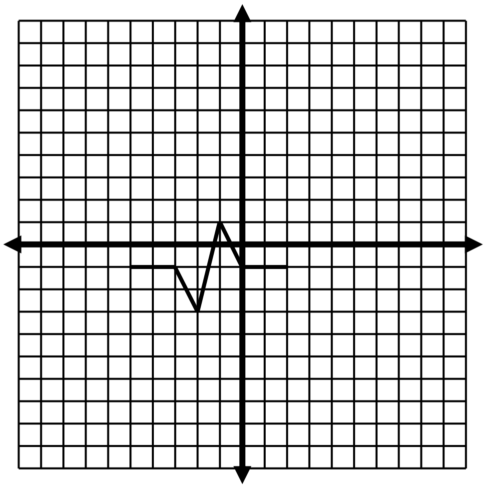
2. Apply the transformations listed above to each of the four characteristic points.

3. Complete the graph of Dipsy using your new characteristic points from #2.

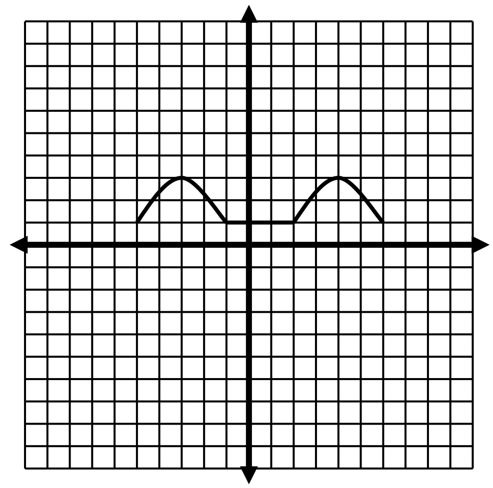
Transformations with Fred Functions

IV. Now that we have practiced transformational graphing with Dipper and his children, you and your partner should use the process learned from the previous three problems to complete the following.

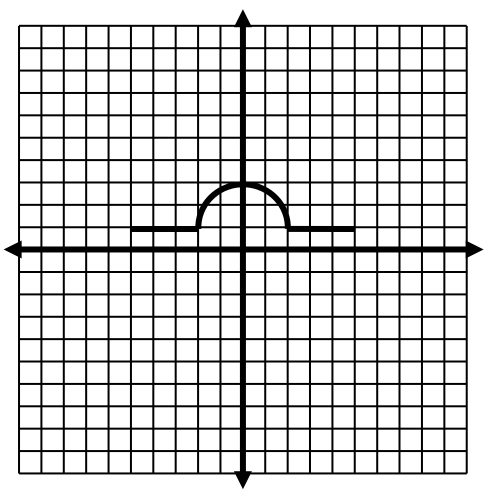
1. Given Cardio, C(x), graph: **$y = 3C(x) + 5$**



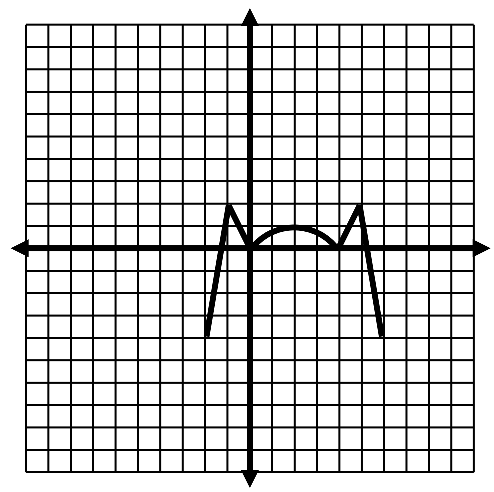
2. Given Garfield, G(x), graph: **$y = -G(x - 3) - 6$**



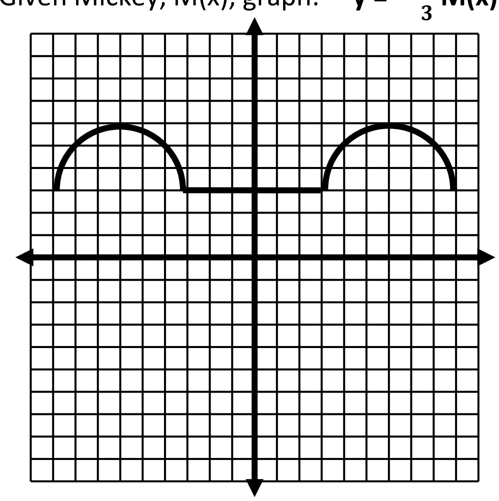
3. Given Horizon, H(x), graph: **$y = -3H(x)$**



4. Given Batman, B(x), graph: **$y = B(-x) + 8$**



5. Given Mickey, M(x), graph: **$y = -\frac{1}{3}M(x)$**



V. Finally, let's examine a reflection of Harry in the line y = x.

1. Graph this line (y = x) on the grid.

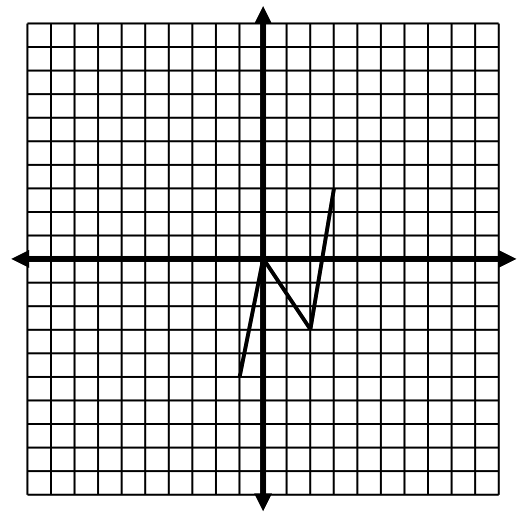
2. Complete the charts below with the characteristic points:

Harry, y = H(x)

x	y

Harry's reflection in y = x:

x	y



3. Compare the points in the two charts. Describe what happens when we reflect in the line y = x.

(This should match what we learned in our earlier study of reflections in the line y = x.)

4. A reflection in the line y = x, shows a graph's **inverse**. We will study this in more depth in a future unit. Look at the graph of Harry's inverse. Is the inverse a function? Explain how you derived your answer.

Transformations with Fred Functions

HW Composite Transformations

1. List the transformations needed to graph the following. Remember that translations are done last.

a. y = 2F(x)+2 ____

b. y = 1/3 F(x-6) ____

c. y = –F(x) – 12 ____

d. y = 3F(-x) ____

e. y = –5F(x) ____

2. Looking back at the examples of parent functions we have worked with, create your own original parent function on the graph. Make sure that you have graphed a function.

a. Explain the name you picked.

b. Write an equation for your function that will have the following effects.

- Stretch vertically by 2 and translate left 4.

  ____

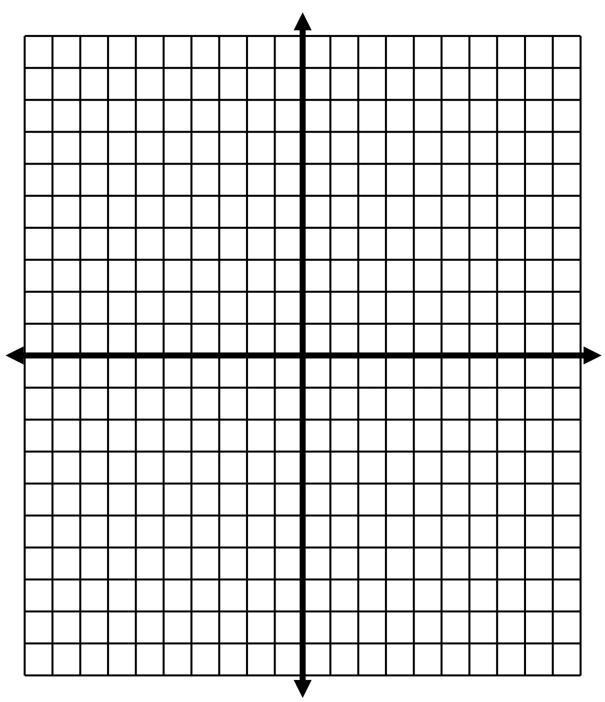
- Translate up 6 and right 4

  ____

- Reflect in the x-axis and compress vertically by ½

  ____

c. Graph each of the children from part c above using a separate graph for each.



d.